

Introduction to the Physics of Correlated Electrons

March 2024

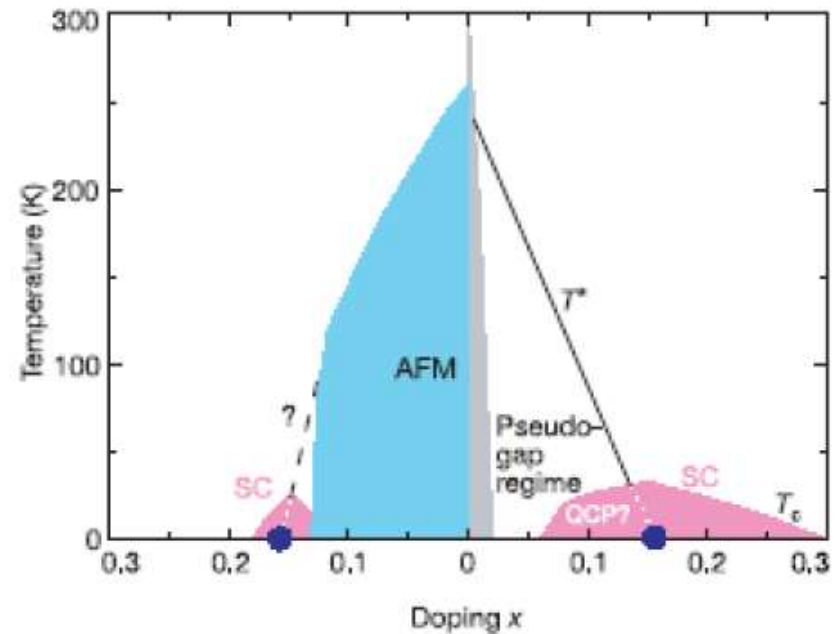
Example: High Temperature Superconductors

L. Alff, *et. al.*, NATURE, VOL 422, p.698 (2003)

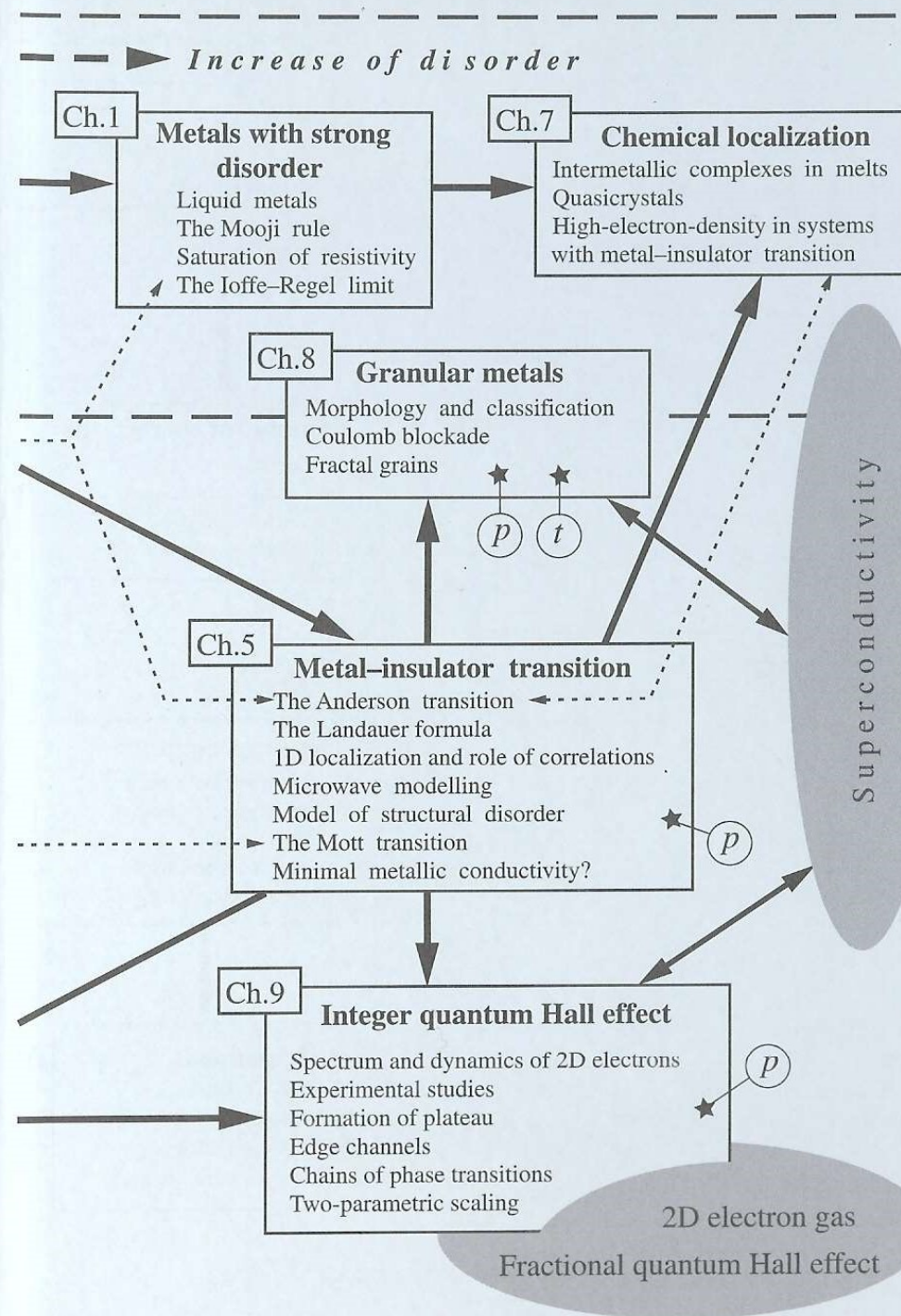
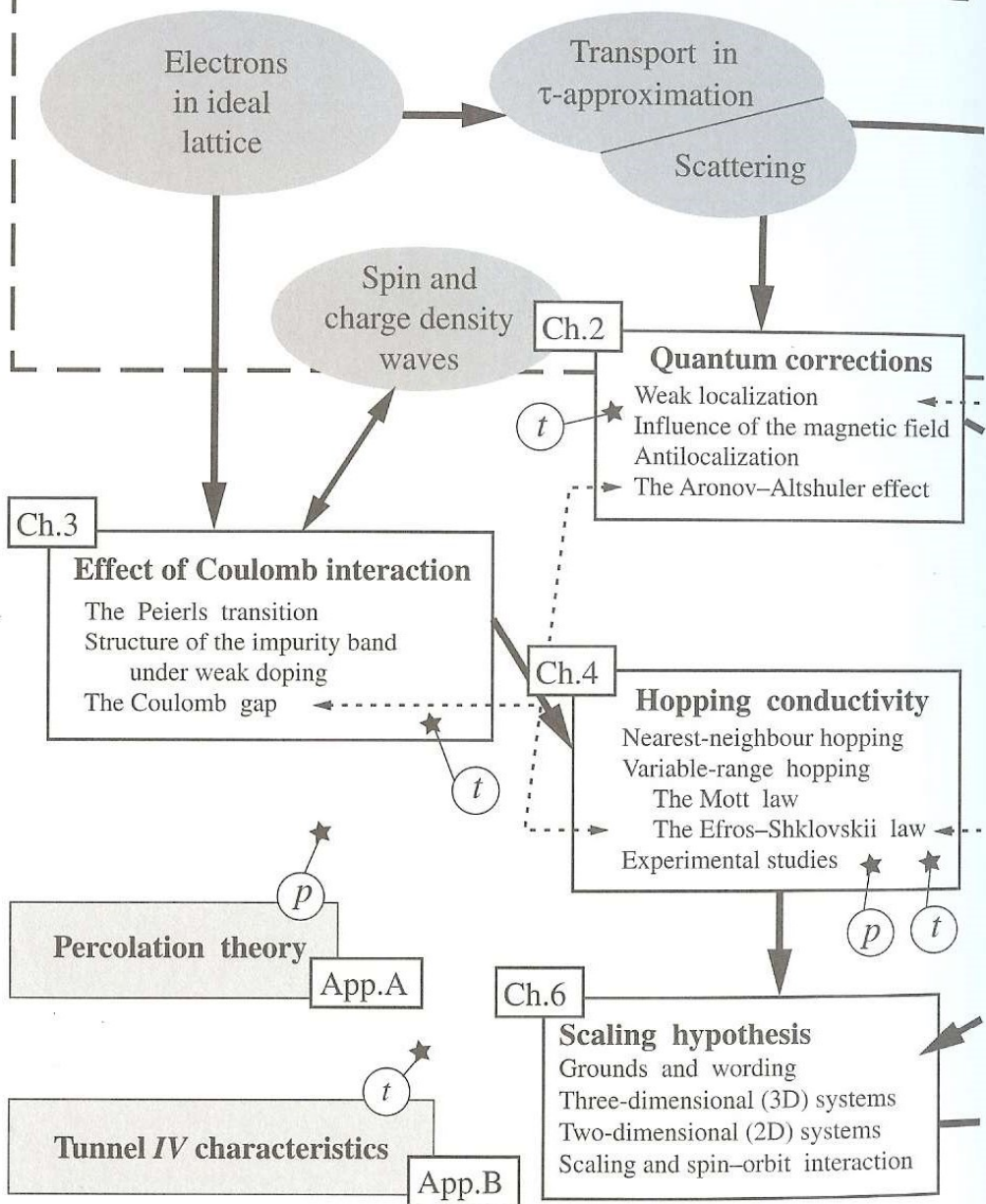
Topics

- metals with strong disorder
- weak localization
- anomalous magnetoresistance
- electron-electron interactions
- hopping conductivity
- Mott-Anderson metal-insulator transitions
- scaling theory of localization

Textbook: V.F. Gantmakher
“*Electrons and Disorder in Solids*”,
Oxford University Press 2005
ISBN 0-19-856756-1



High-density electron gas



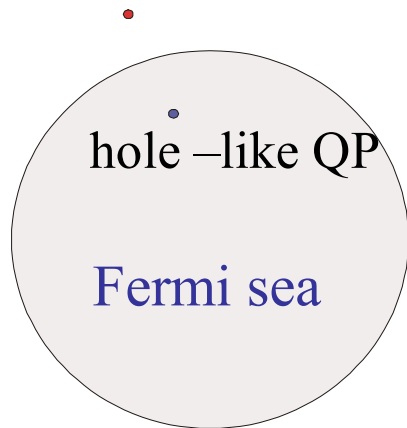
Electrons and disorder in solids
 (Schematic guide)

Fermi liquid and quasiparticles

Quasiparticle concept (Landau 1956, 1957)

Fermi liquid - a system of interacting Fermi particles

electron-like QP



hole-like QP

Fermi sea

Quasiparticle (QP) – excitation in a Fermi liquid, it resembles an excitation in an ideal Fermi gas, but not equivalent

Due to interaction with other electrons and ions, quasiparticle **effective mass** m^* differs from the free electron mass m_e

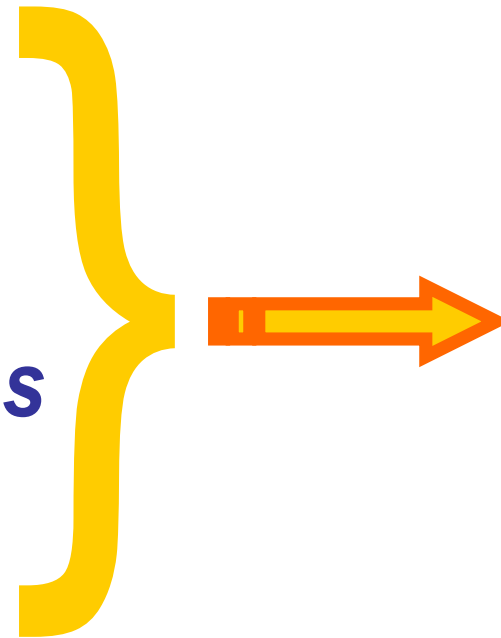
In superconductors **effective charge** e^* also differs from electronic charge e

Excitation energy $\xi = p^2 / 2m^* - p_F^2 / 2m^* \approx v_F (p - p_F)$

Quasiparticles have finite **lifetime** due to interaction with other electrons, phonons, etc.

Fermi Liquid

- *Fermi statistics*
- *Low temperatures*
- *Not too strong interactions*
- *Translation invariance*



*Fermi
Liquid*

L.D. Landau, "Fermi-Liquid Theory"
Zh. Exp. Teor. Fiz., v.30, p.1058 (1956)

- *Fermi statistics*
- *Low temperatures*
- *Not too strong interactions*
- *Translation invariance*



*Fermi
Liquid*

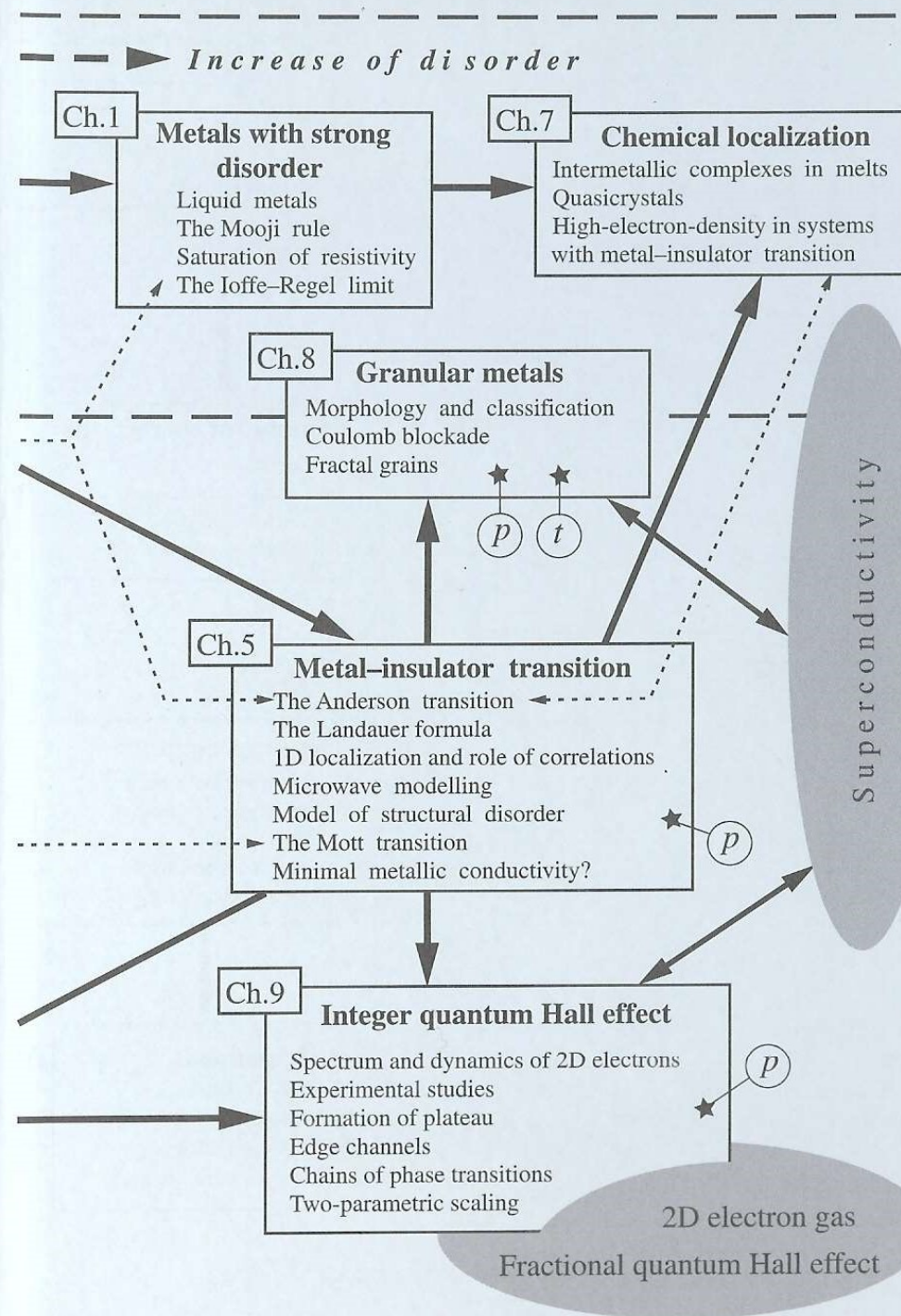
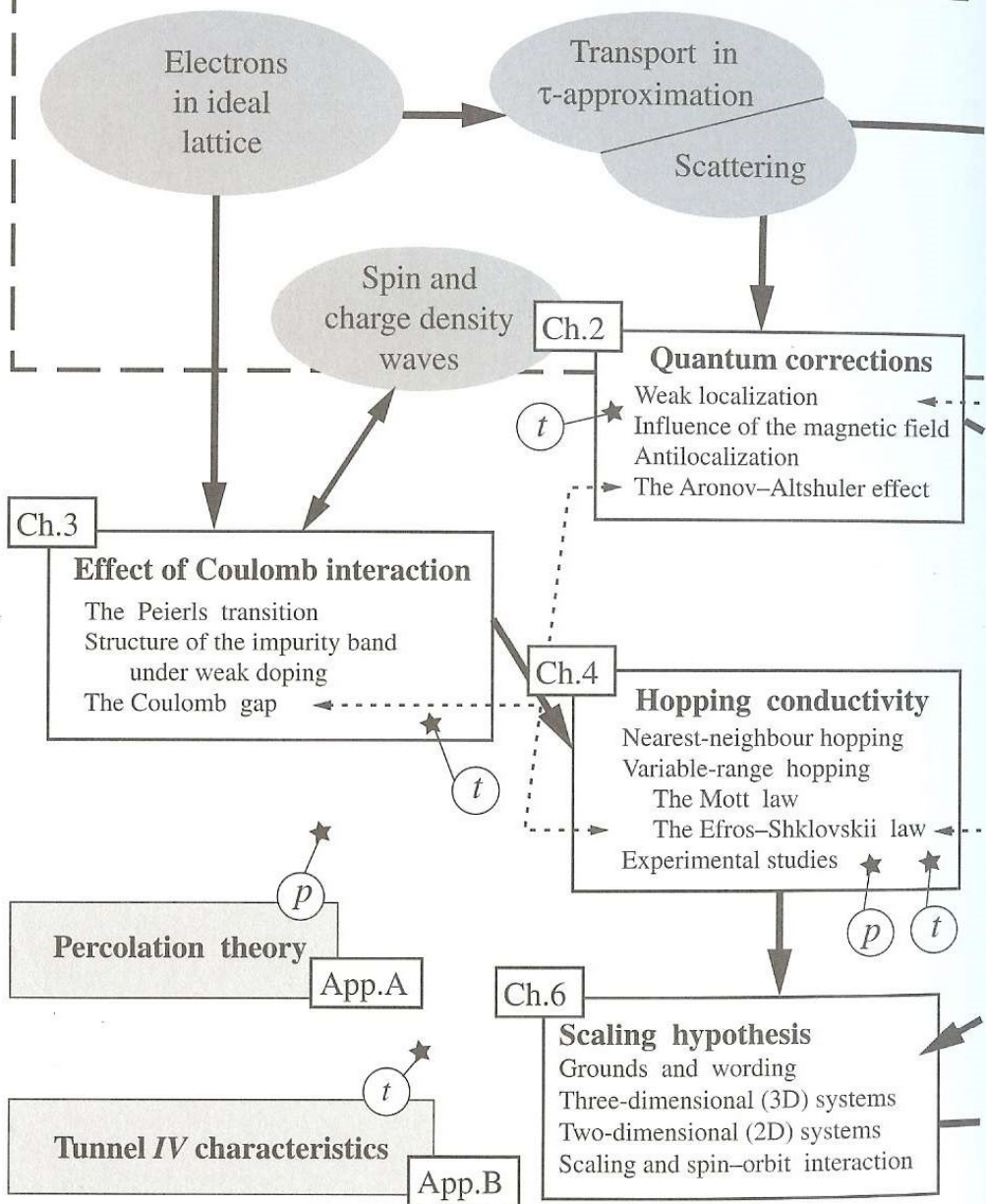
It means that

1. *Excitations are similar to the excitations in a Fermi-gas:*
 - a) *the same quantum numbers – momentum, spin $\frac{1}{2}$, charge e*
 - b) *decay rate is small as compared with the excitation energy*
2. *Substantial renormalizations. For example, in a Fermi gas*

$$\partial n / \partial \mu, \quad \gamma = c / T, \quad \chi / g \mu_B$$

*are all equal to the one-particle density of states ν .
These quantities are different in a Fermi liquid*

High-density electron gas

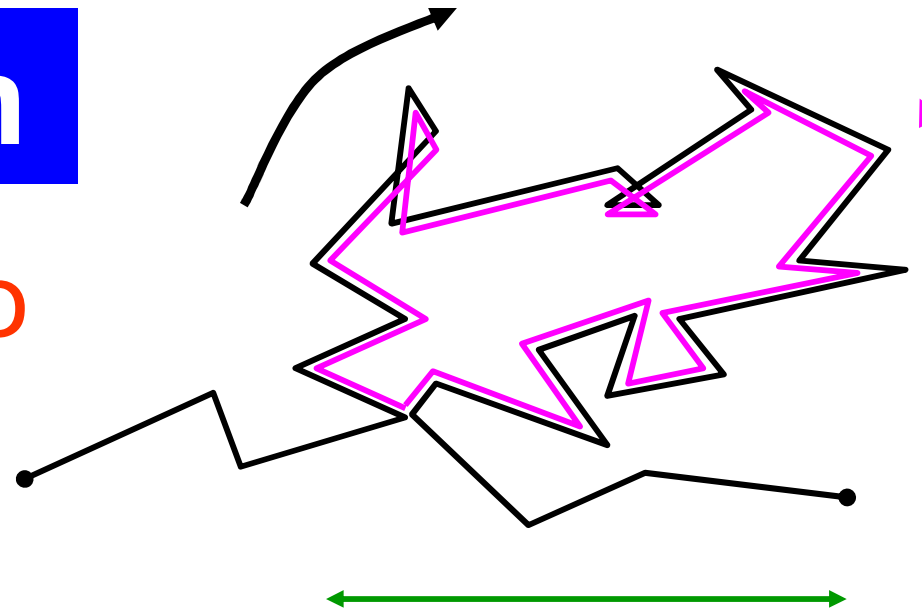


Electrons and disorder in solids
 (Schematic guide)

Weak Localization

Quantum corrections to conductivity

$$\sigma = \frac{ne^2 l}{\hbar k_F}$$



Two types of electronic scattering

elastic scattering, probability $1/\tau$

inelastic scattering, probability $1/\tau_\phi$

Phase ϕ of the wave
function

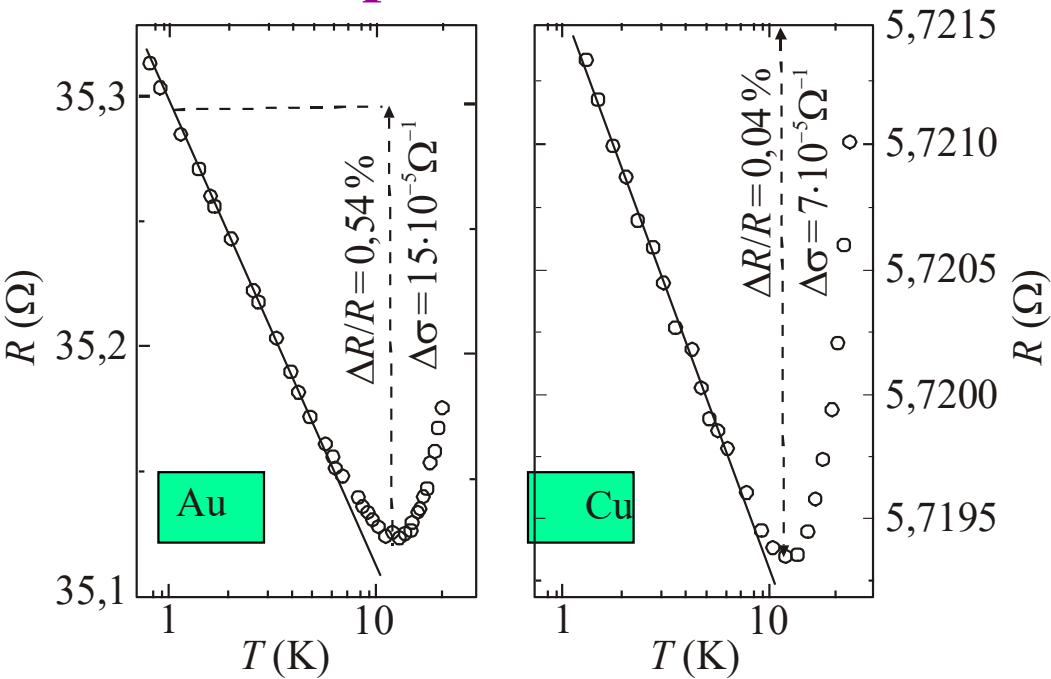
$$\psi \propto \exp(i\varepsilon t / \hbar)$$

$$\phi = \varepsilon t / \hbar$$

$$\tau \ll \tau_\phi$$

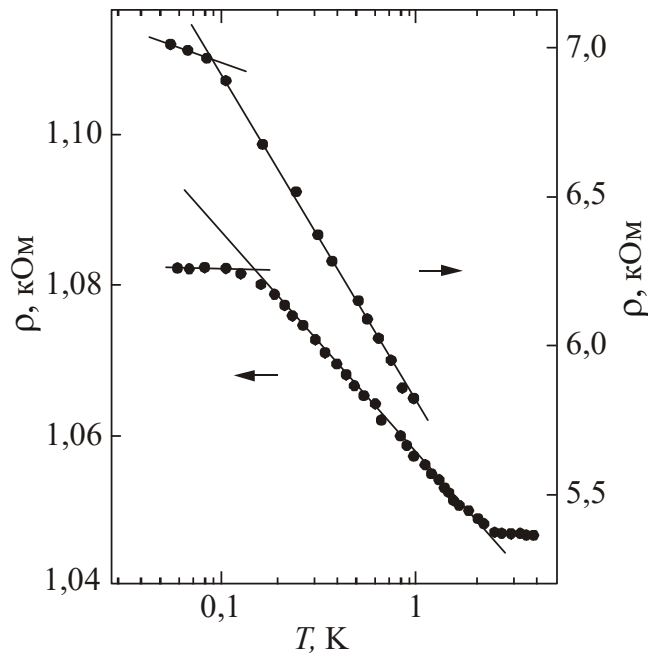
- phase coherence

Experiment



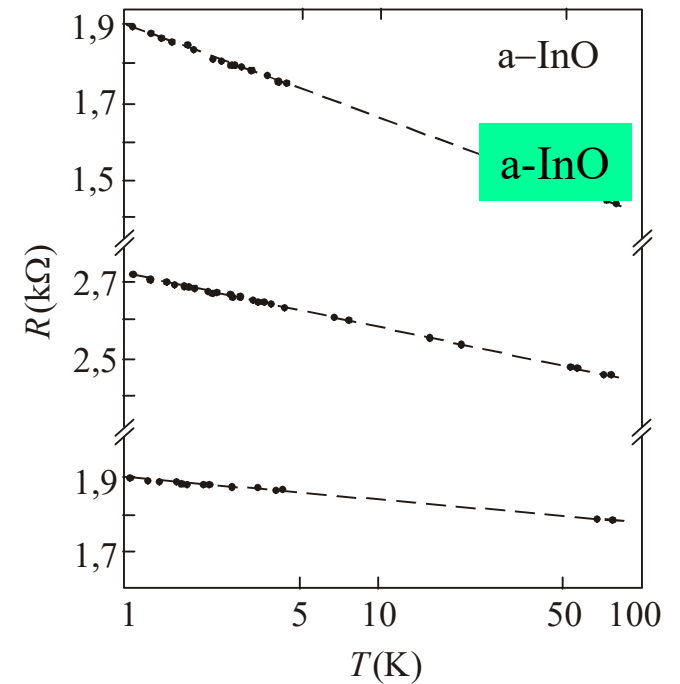
Cu – van der Dreis *et al.*, PRL **46**, 565 (1981)

Au – S. Dorozhkin, JETP Lett. **36**, 15 (1982)



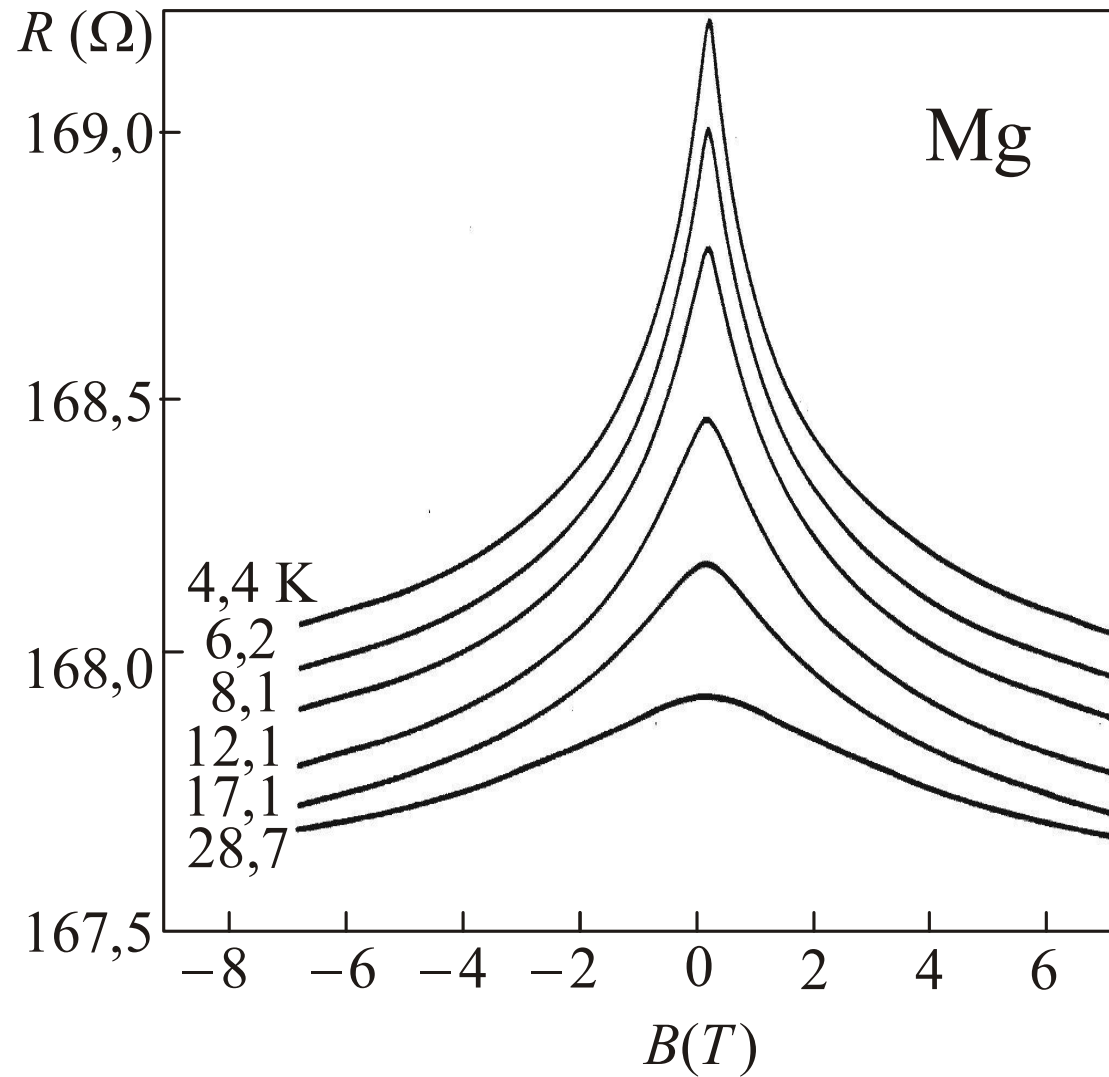
D.J.Bishop, D.C.Tsui,
R.C.Dynes, PRL **44**,
1153 (1980)

2DEG heterostructures



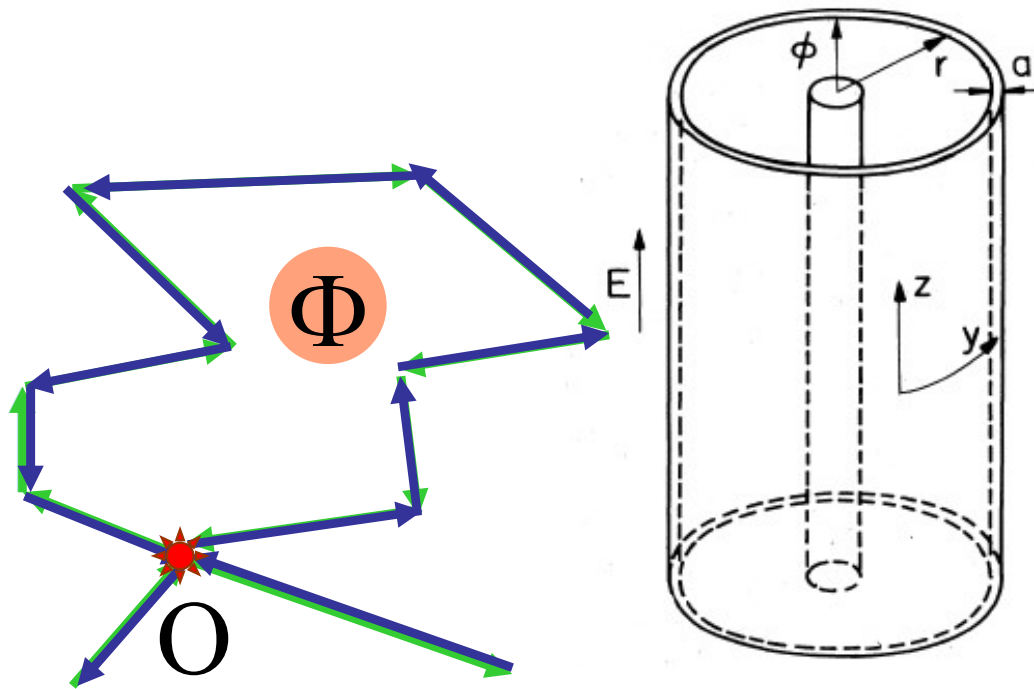
Z.Ovadyahu, Y.Imry
PRB **24**, 7440 (1981)

Anomalous (negative) magneto-resistance



G. Bergmann, Phys.Rep. **107**, 1 (1981)

Aharonov-Bohm effect



With magnetic field H

$$\varphi_1 - \varphi_2 = 2\pi \Phi / \Phi_0$$

Resistance is a periodic function of the magnetic flux with the period

$$\Phi_0 = h/2e$$

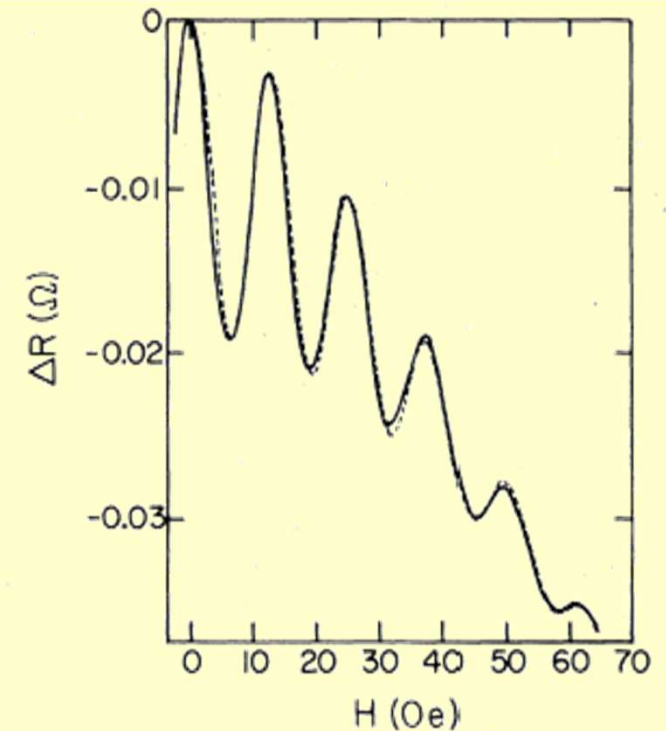


FIG. 8. Longitudinal magnetoresistance $\Delta R(H)$ at $T=1.1$ K for a cylindrical lithium film evaporated onto a 1-cm-long quartz filament. $R_{4,2}=2$ k Ω , $R_{300}/R_{4,2}=2.8$. Solid line: averaged from four experimental curves. Dashed line: calculated for $L_{\varphi}=2.2$ μm , $\tau_{\varphi}/\tau_{s0}=0$, filament diameter $d=1.31$ μm , film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d=1.30\pm 0.03$ μm (Altshuler *et al.*, 1982; Sharvin, 1984).

Metals with strong electronic scattering

$$\sigma = \frac{ne^2l}{\hbar k_F}$$

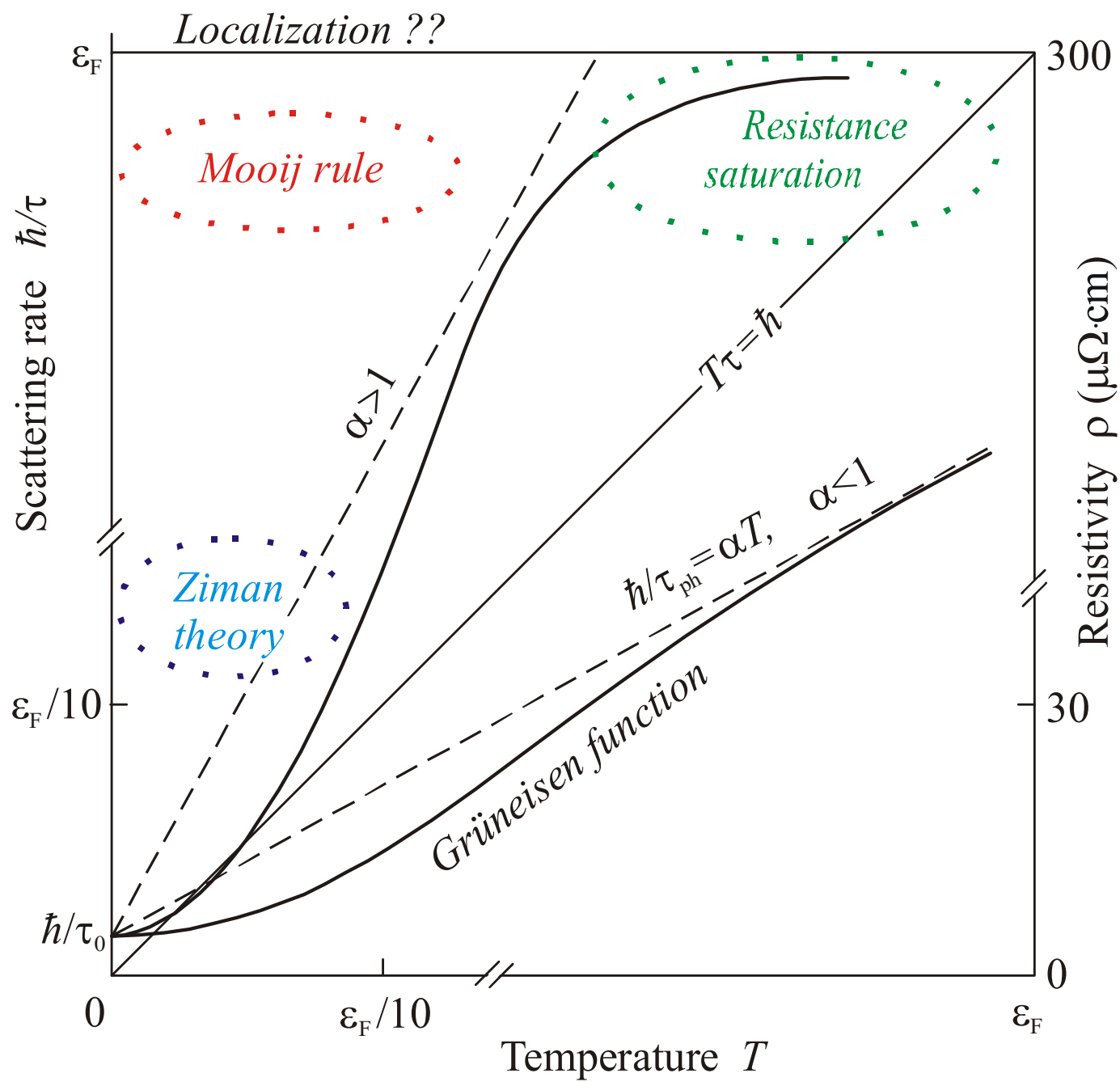
$k_F l > 1$ – Ioffe-Regel rule

$$k_F = (3\pi^2 n)^{1/3}$$

$$\sigma = A \frac{e^2}{\hbar} n^{1/3} (k_F l)$$

$$n^{-1/3} \approx 3 \text{ \AA}$$

$$\rho_{\max} \sim (100 \div 1000) \mu\Omega \cdot \text{cm}$$

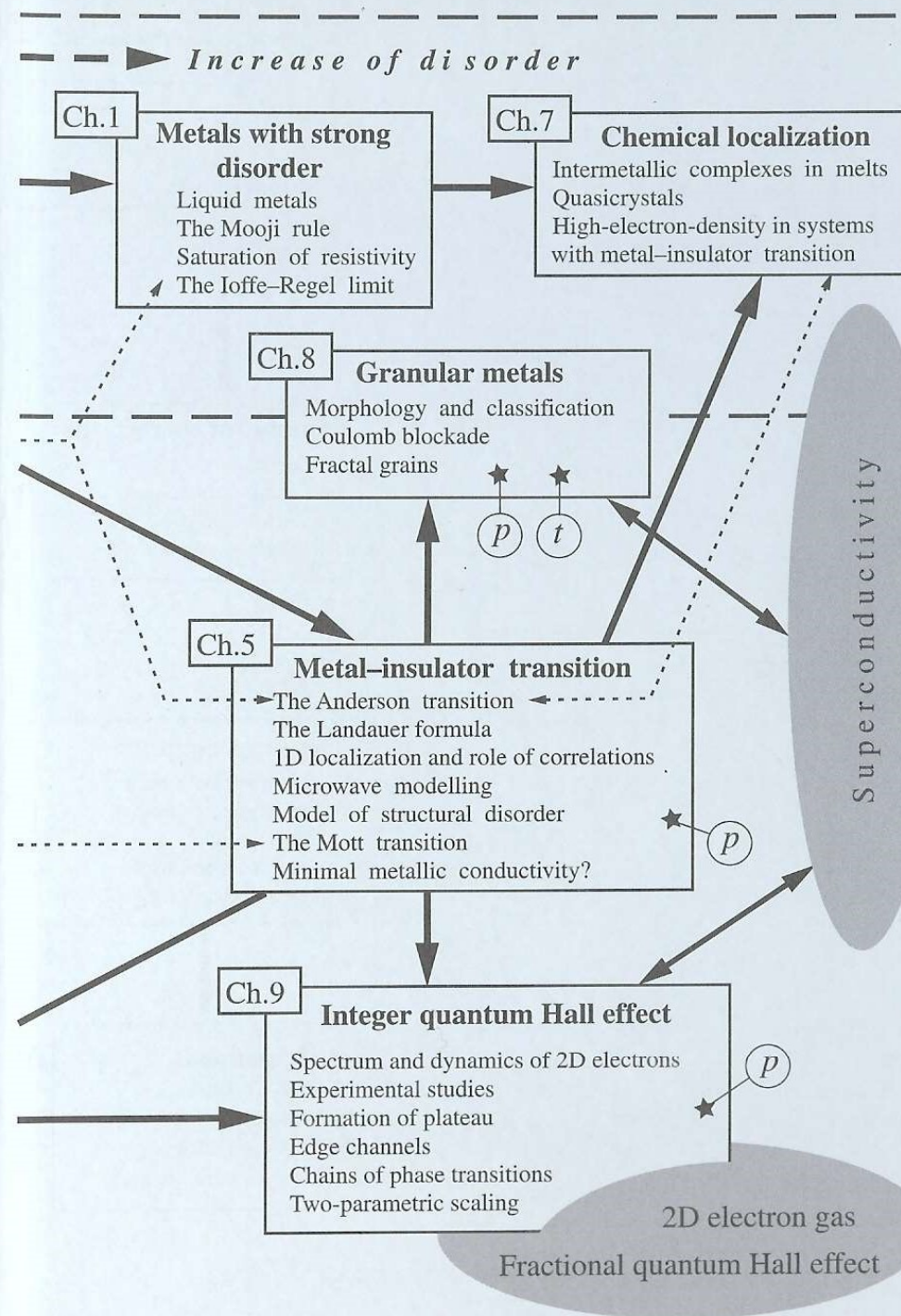
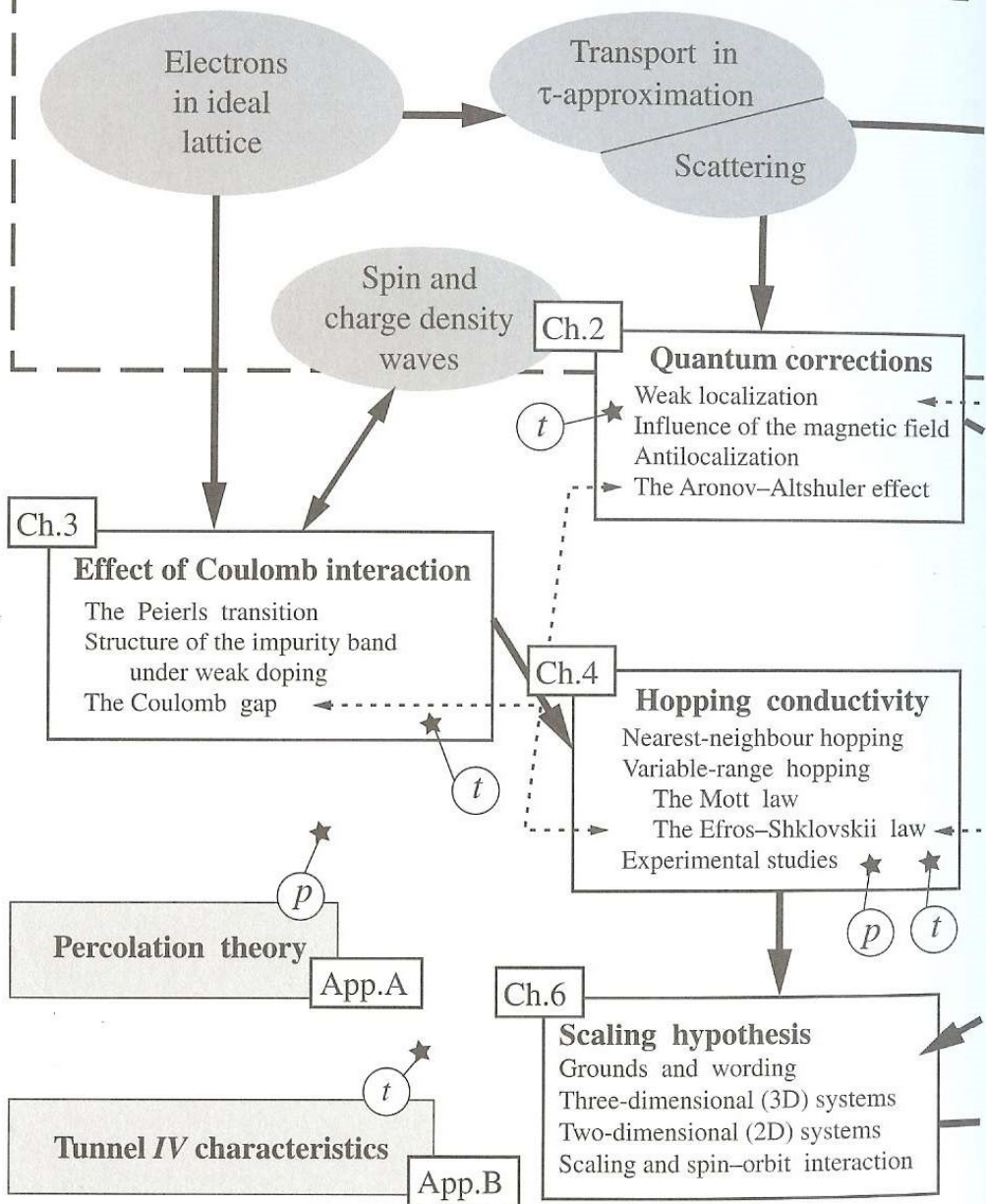


α (experiment.)

K	1.75
Cu	1.35
Al	0.4
Sn	0.07
W	0.2
Pb	0.1
Ag	1.5

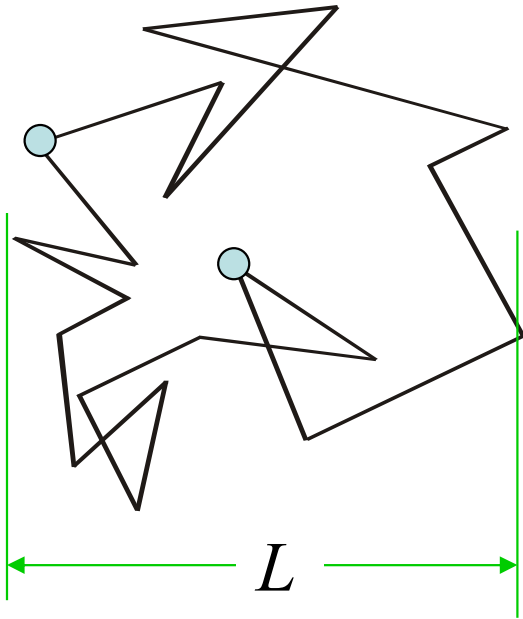
Effects of Coulomb Interaction

High-density electron gas



Electrons and disorder in solids
(Schematic guide)

ee - interaction (interference)



Ballistic regime

$$r \sim v_F t$$

Diffusion regime

$$r \sim l \sqrt{t/\tau} \sim v_F \sqrt{t\tau}$$

Phase $\exp(i\varphi) = \exp[i(\epsilon_i/\hbar)t]$, $\Delta\varphi = (\Delta\epsilon/\hbar)t$

Dephasing time

$$\left. \begin{array}{l} \tau_{ee} \sim \hbar/\Delta\epsilon \\ \Delta\epsilon \sim T \end{array} \right\}$$

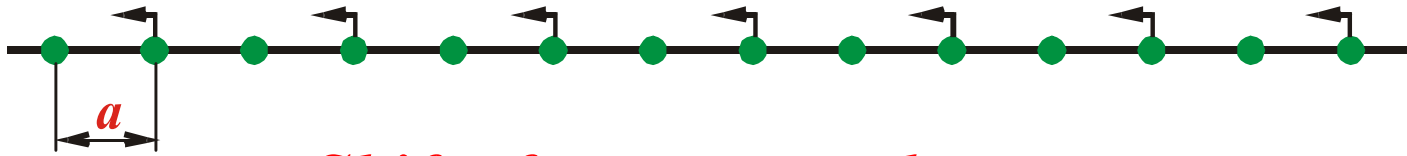
$$\tau_{ee} \sim \hbar/T$$

Dephasing length

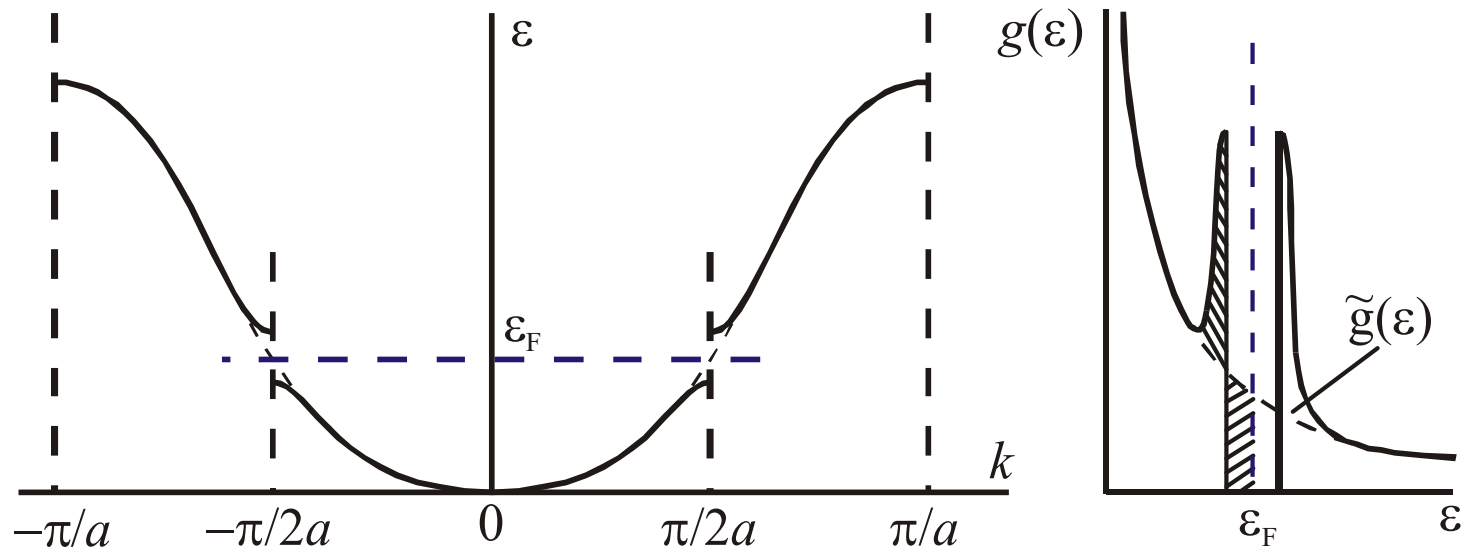
$$L_{ee} \approx l \sqrt{\frac{\tau_{ee}}{\tau}} \approx v_F \sqrt{\frac{\hbar\tau}{T}} \approx \sqrt{\frac{\hbar D}{T}}$$

Diffusing electrons keep coherence during time τ_{ee} keeping the typical distance L_{ee} .

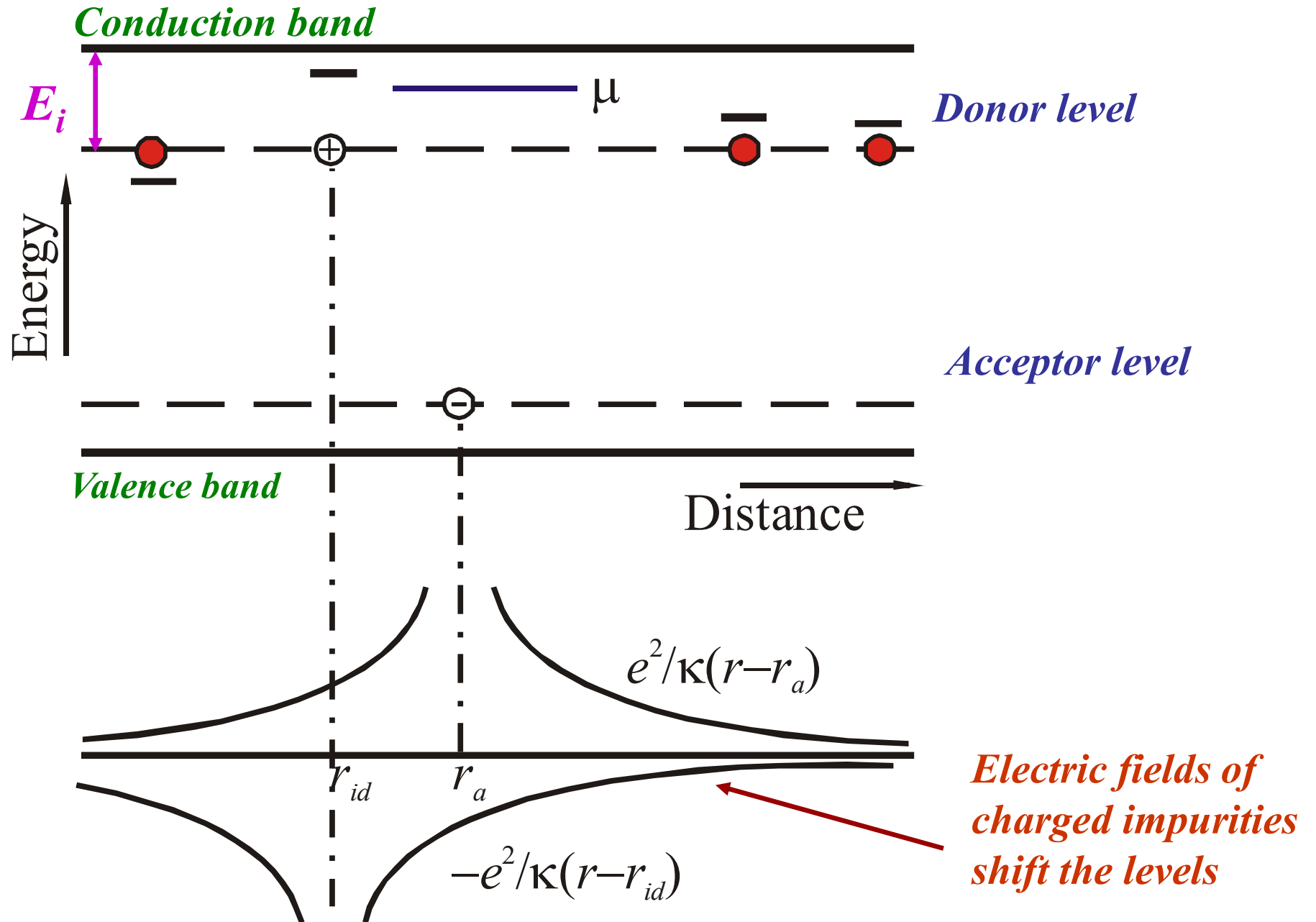
Peierls transition



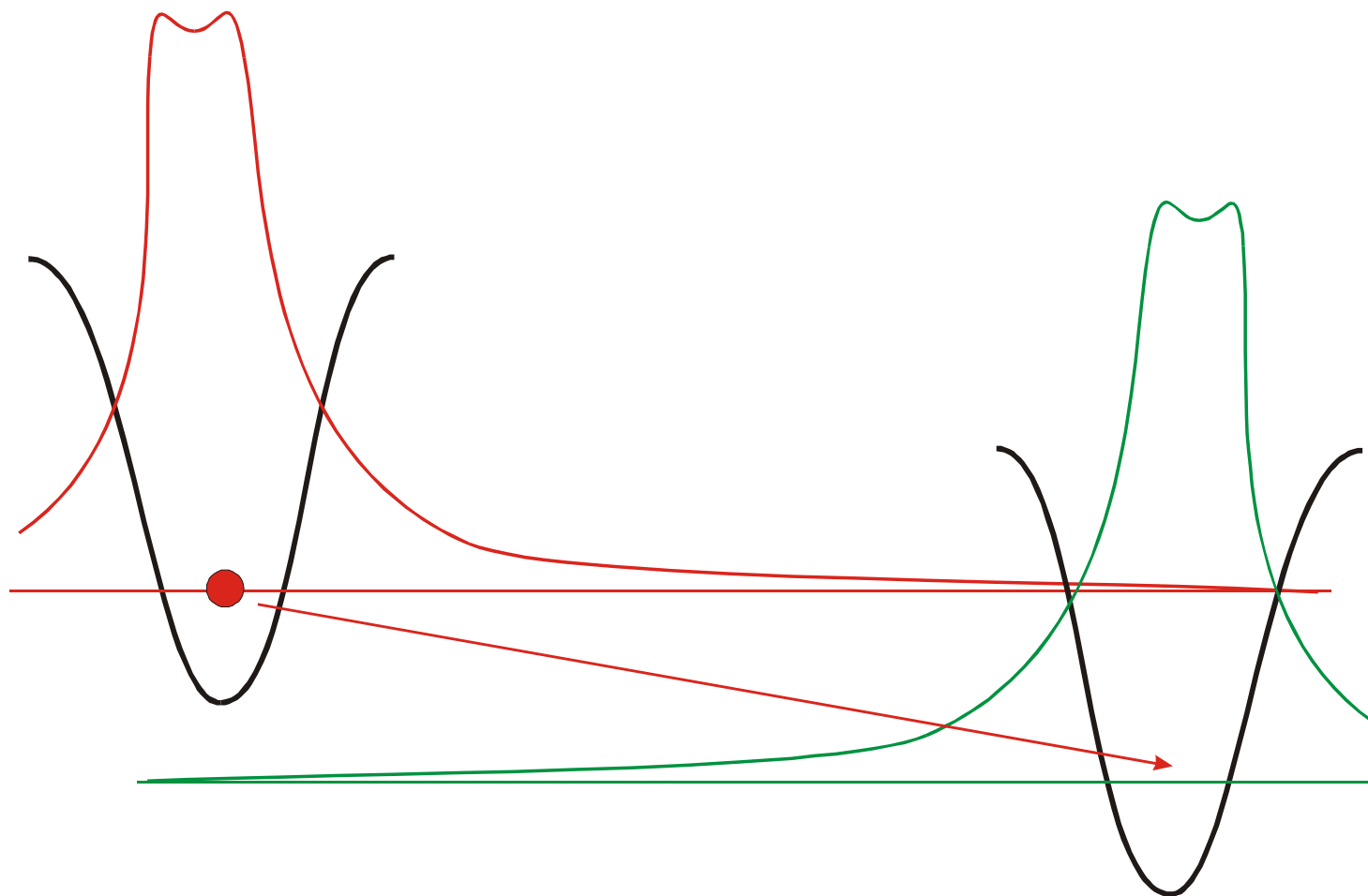
*Shift of every second atom \Rightarrow
period doubling $a \rightarrow 2a \rightarrow$*



Impurity band

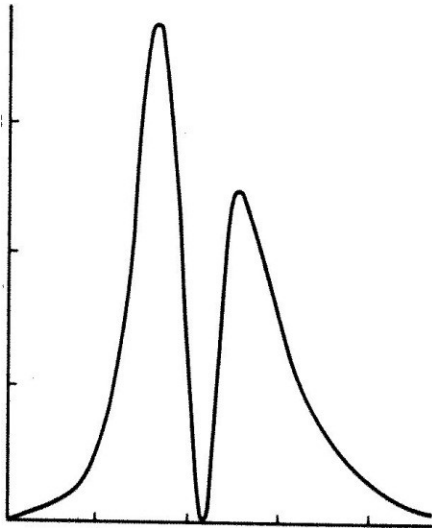
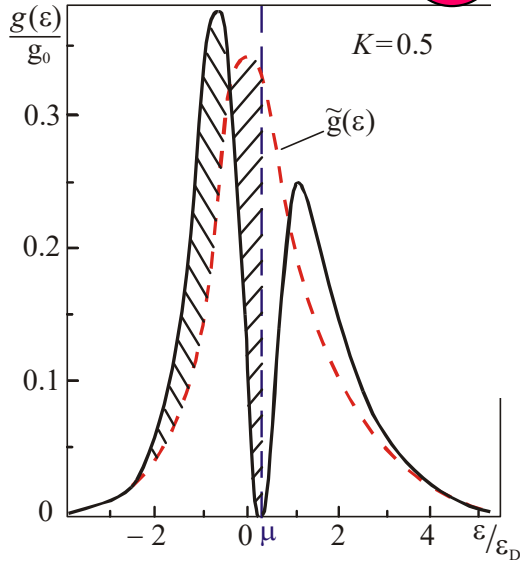


Hopping conductivity

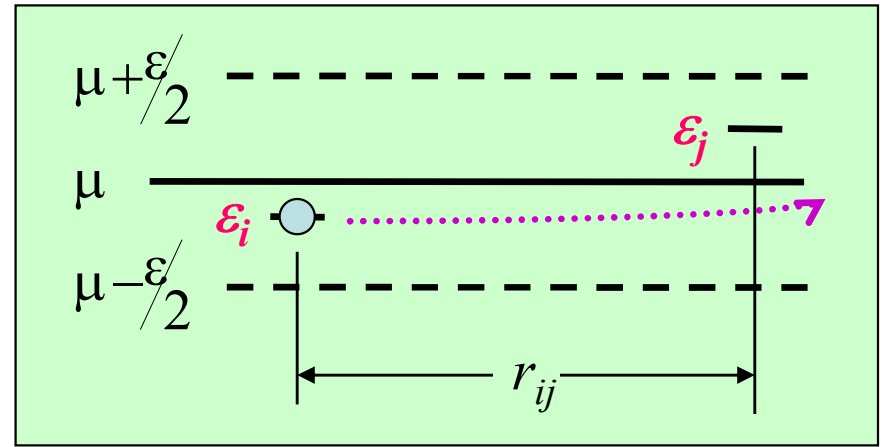


“Coulomb gap”

I



II



Energy $\epsilon_j - \frac{e^2}{\kappa r_{ij}}$ should be larger than ϵ_i

$$\epsilon_j - \epsilon_i - \frac{e^2}{\kappa r_{ij}} \geq 0 \rightarrow \frac{e^2}{\kappa r_{ij}} < \epsilon$$

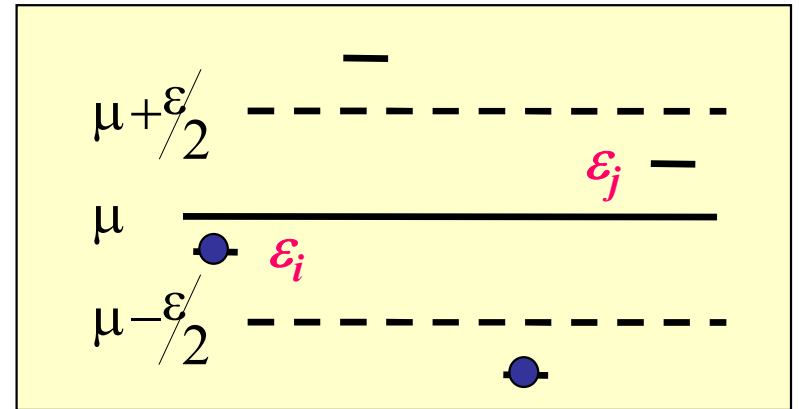
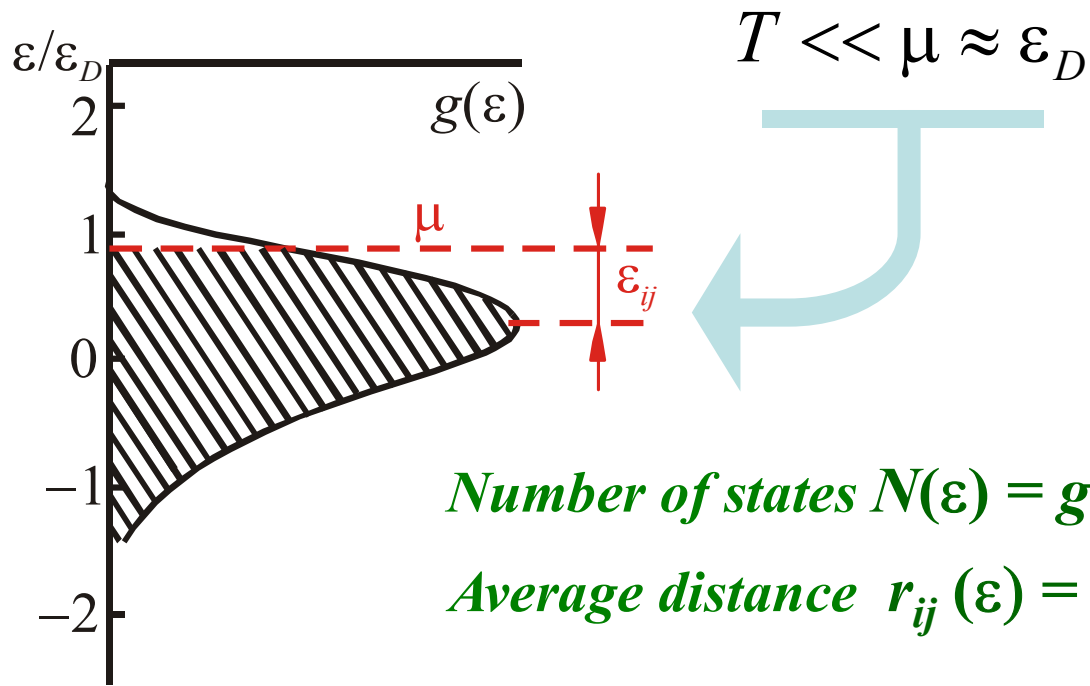
3D

$$N(\epsilon) = r_{ij}^{-3} \leq \left(\frac{\kappa \epsilon}{e^2} \right)^3, \quad g(\epsilon) = \frac{\partial N}{\partial \epsilon} \propto \frac{(\epsilon - \mu)^2 \kappa^3}{e^6}$$

2D

$$N(\epsilon) = r_{ij}^{-2} \leq \left(\frac{\kappa \epsilon}{e^2} \right)^2, \quad g(\epsilon) = \frac{\partial N}{\partial \epsilon} \propto \frac{|\epsilon - \mu| \kappa^2}{e^4}$$

Variable range hopping: Mott low



Number of states $N(\varepsilon) = g_\mu \varepsilon$

Average distance $r_{ij}(\varepsilon) = [N(\varepsilon)]^{-1/3}$,

Energy difference $\sim \varepsilon$

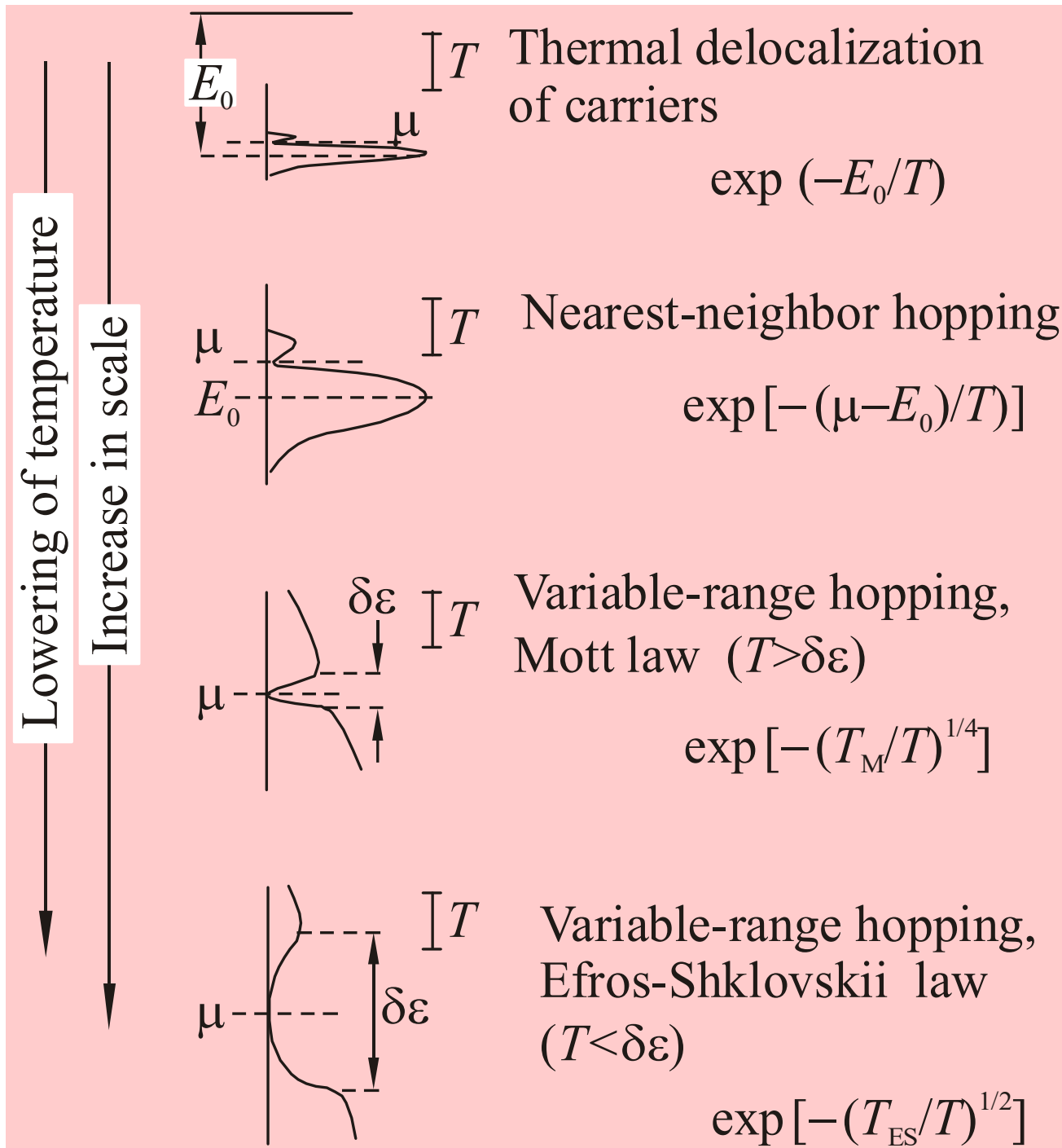
Parameter u_{ij} *of Abrahams-Miller network*

$$u_{ij} = \frac{2}{a_B [N(\varepsilon)]^{1/3}} + \frac{\varepsilon}{T} = \frac{2}{g_\mu^{1/3} a_B \varepsilon^{1/3}} + \frac{\varepsilon}{T}$$

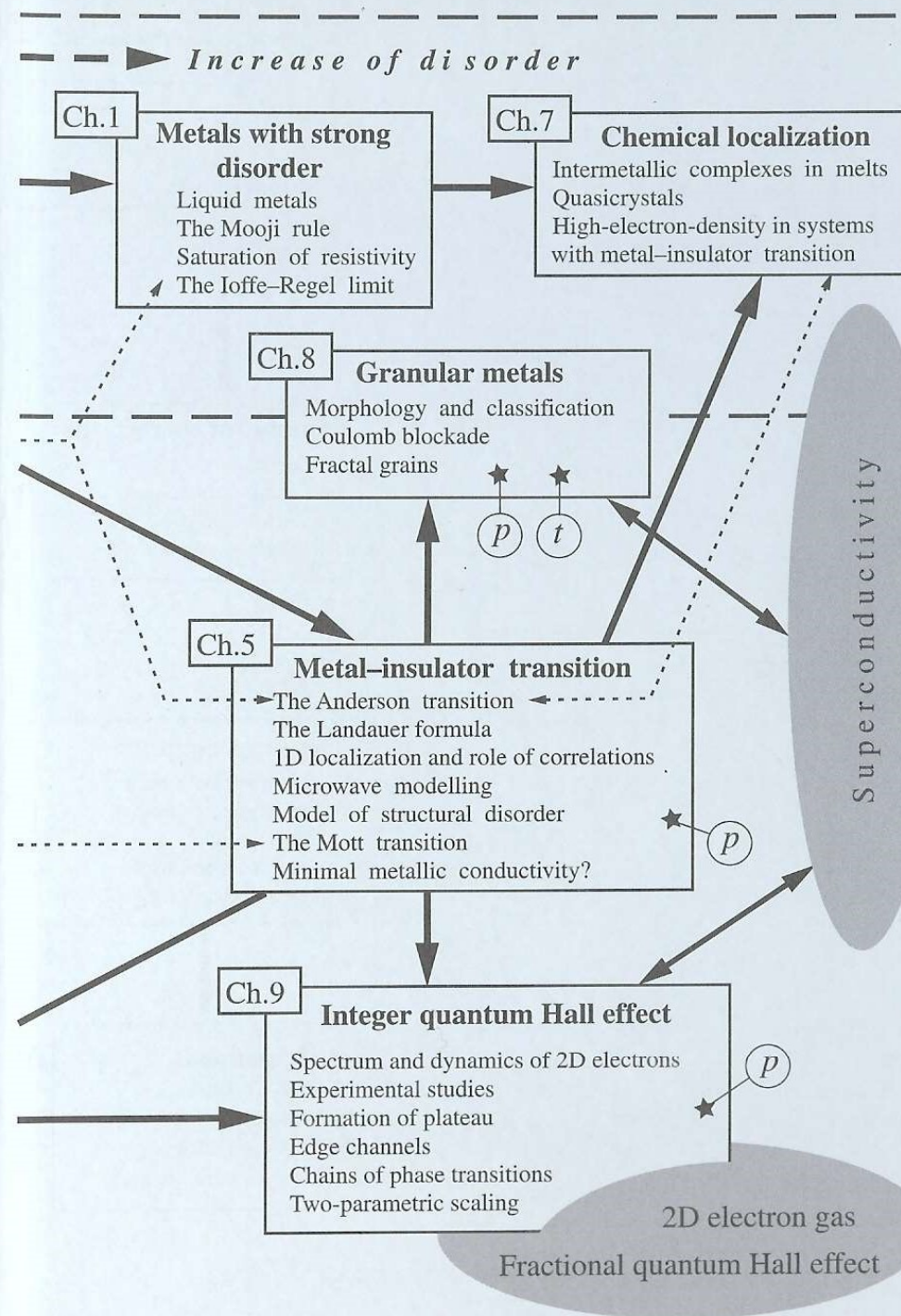
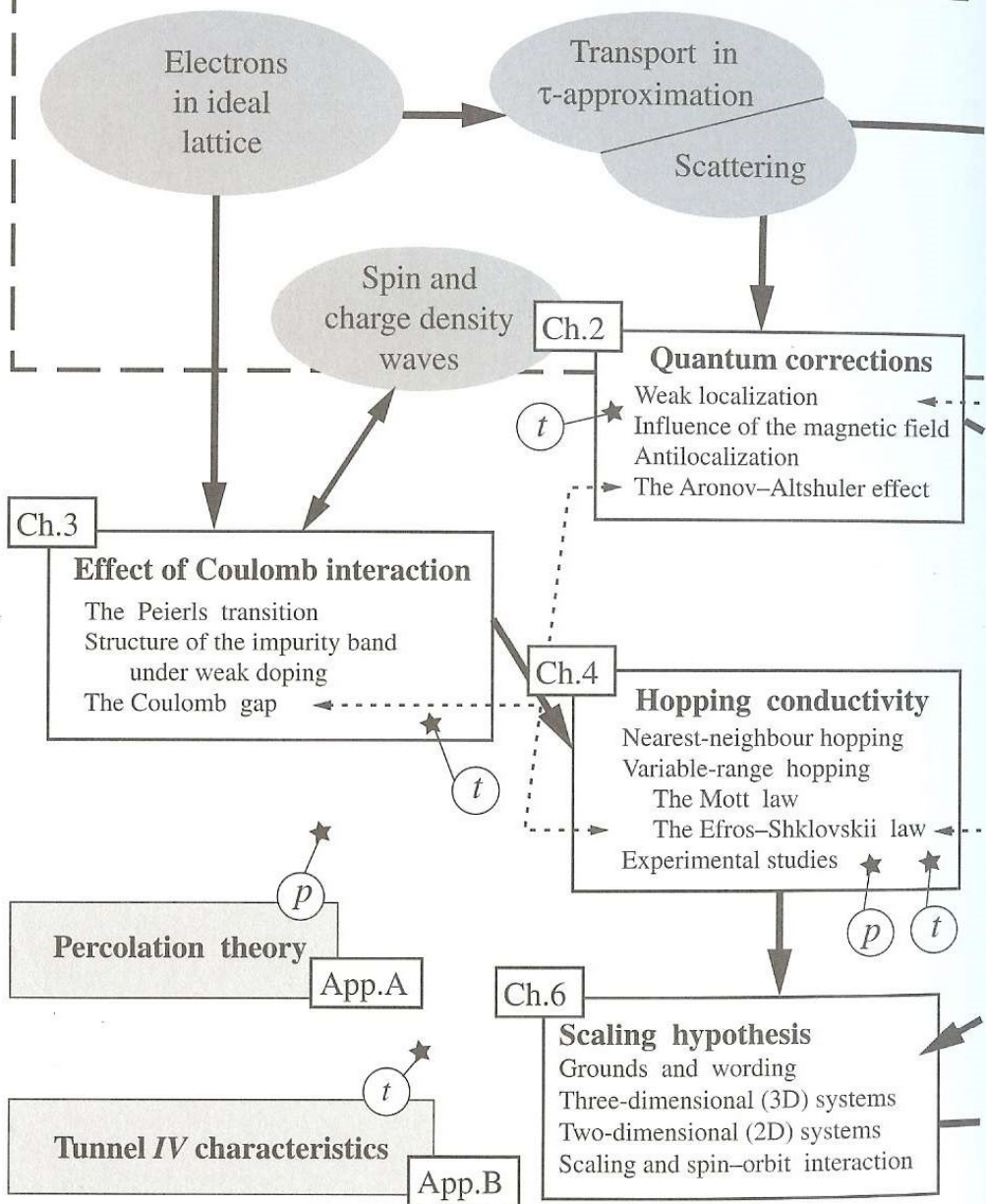
u_{ij} has a maximum when $\frac{d}{d\varepsilon} u_{ij}(\varepsilon) = 0$,

$$\Rightarrow \varepsilon_{\min} = \left(\frac{T}{a_B g_\mu^{1/3}} \right)^{3/4} = (T^3 T_{Mott})^{1/4}, \quad T_{Mott} = (a_B^3 g_\mu)^{-1}$$

Mechanisms of hopping conductivity



High-density electron gas



Electrons and disorder in solids
(Schematic guide)

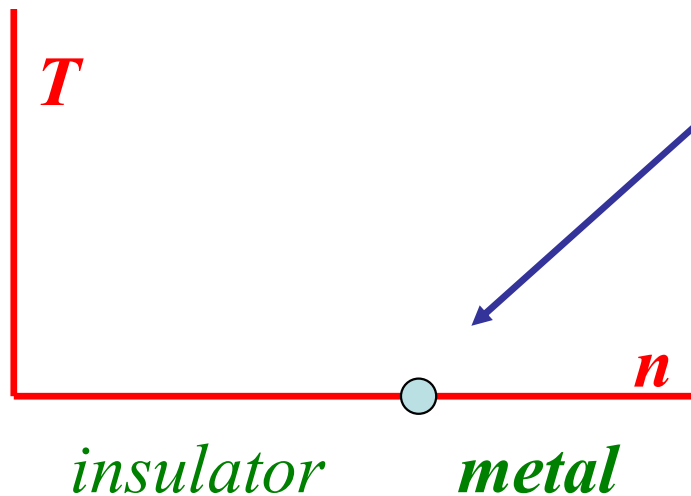
Metal-insulator transitions

*The definition: metal – $\sigma \neq 0$
insulator – $\sigma = 0$*



has a sense only at $T = 0$

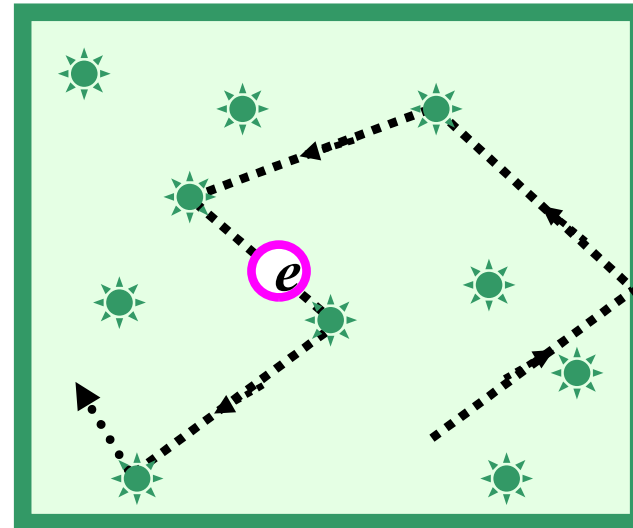
*Isolated point at the
phase diagram*



Anderson localization

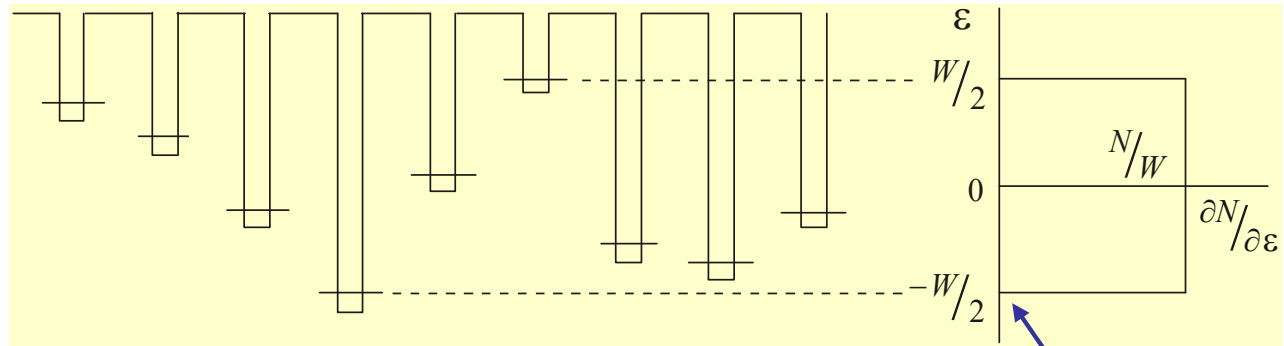


Quantum particle in **random**
quenched potential



Anderson transition

$$\frac{\partial N}{\partial \varepsilon} = \begin{cases} N/W & \text{for } |\varepsilon| < W/2 \\ 0 & \text{for } |\varepsilon| > W/2 \end{cases}$$



Overlap integral

$$J = \int \psi_1^* \hat{H} \psi_2 d^3 r \propto \exp\left(-\frac{r_{12}}{a_B}\right) \equiv \exp\left(-\frac{1}{a_B n^{1/3}}\right) \quad \text{width } W \text{ characterising disorder}$$

ratio J/W - the main parameter of the problem

$$\frac{J}{W} < \left(\frac{J}{W}\right)_{crit}$$

$$\frac{J}{W} > \left(\frac{J}{W}\right)_{crit}$$

insulator

$$\left(\frac{J}{W}\right)_{crit}$$

metal

$$\frac{J}{W}$$

Delocalized states first appear at the energy band center

Mott transition

3 length scales:

$$n^{-1/3}$$

average e-e distance

$$a_B = \frac{\kappa \hbar^2}{m^* e^2}$$

Bohr radius

$$r_e = \left(\frac{4m^* e^2 n^{1/3}}{\kappa \hbar^2} \right)^{-1/2}$$

screening length

are related

$$r_e = \frac{1}{2} \left(a_B n^{-1/3} \right)^{1/2}$$

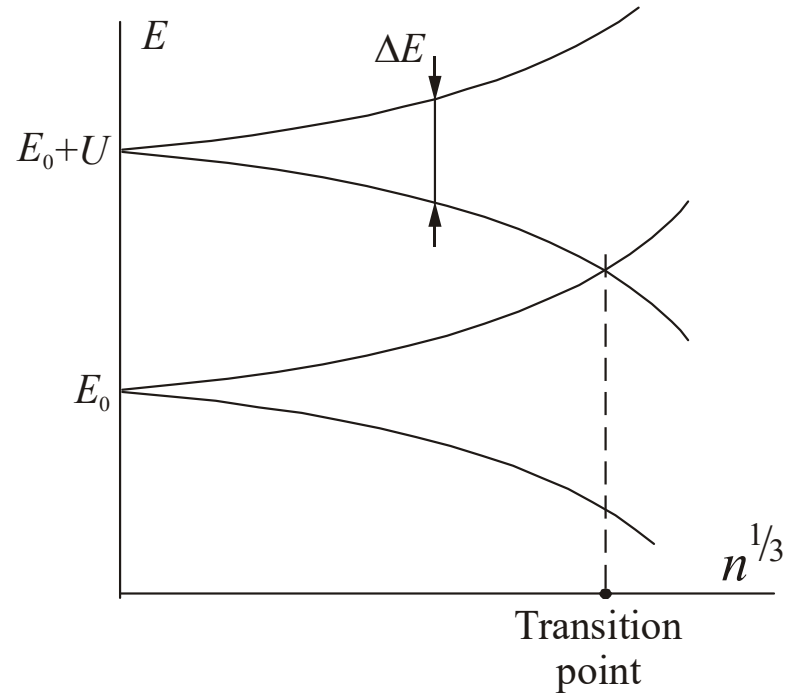
$r_e > a_B$ – insulator

$r_e < a_B$ – metal

Mott transition occurs if

$$r_e = a_B$$

$$a_B n_c^{1/3} = 0.25$$



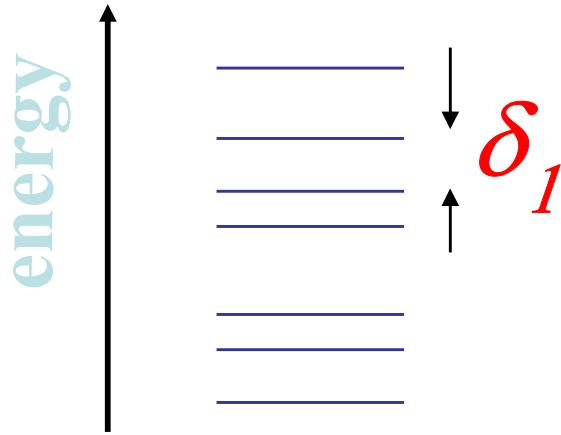
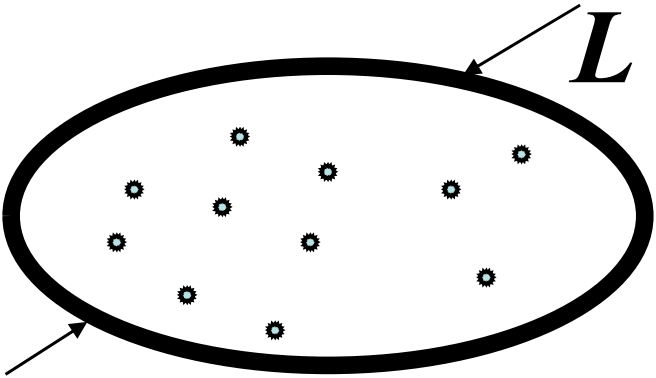
Hubbard model

Energy scales (Thouless, 1972)



1. Mean level spacing

$$\delta_1 = 1/v \times L^d$$



L is the system size;
 d is the number of dimensions

2. Thouless energy

$$E_T = hD/L^2$$

D is the diffusion constant

E_T has a meaning of the *inverse diffusion time* of the traveling through the system or the *escape rate* (for open systems)

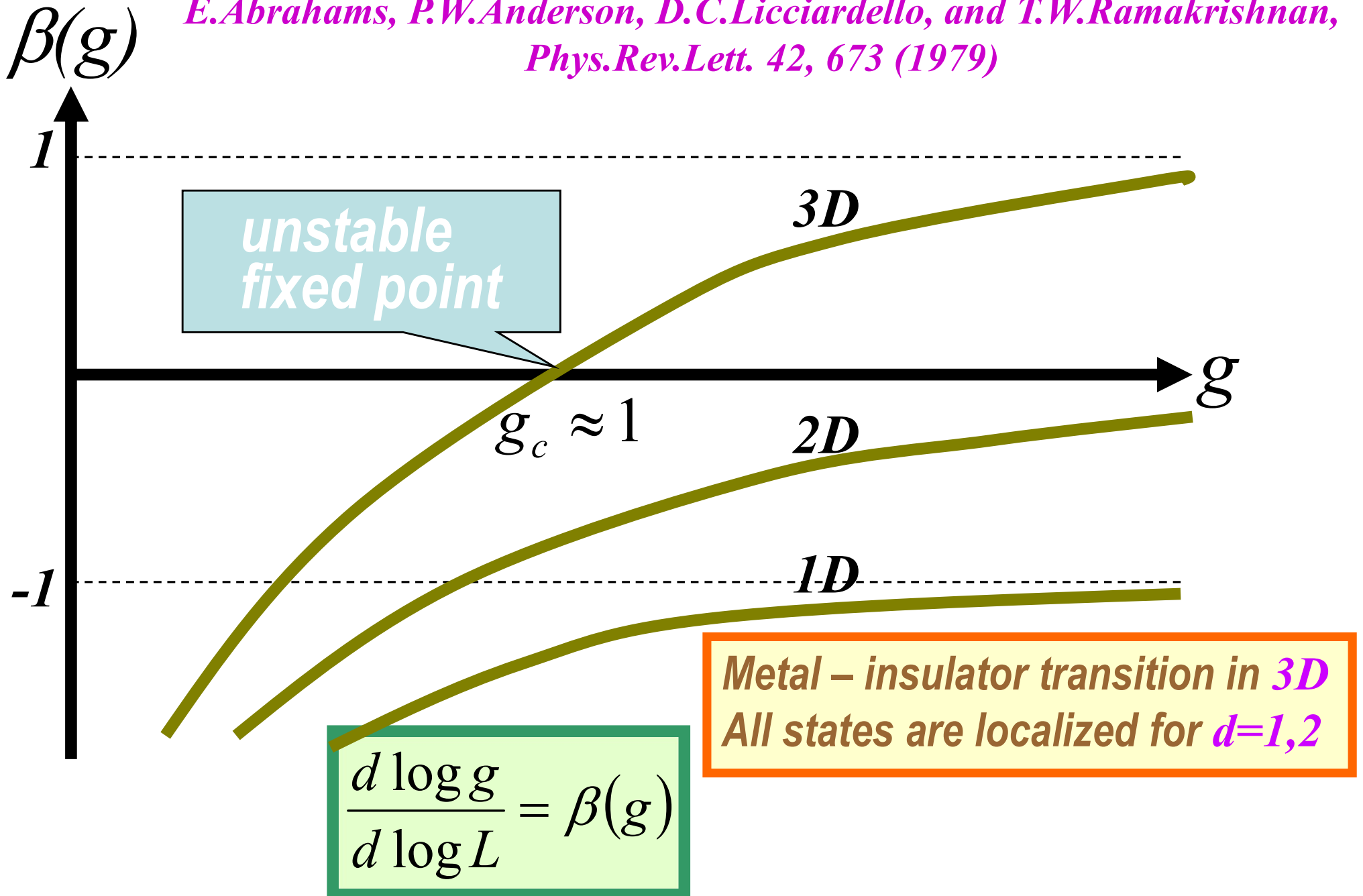
$$g = E_T / \delta_1$$

dimensionless
Thouless
 conductance

$$g = Gh/e^2$$

Scaling theory of localization

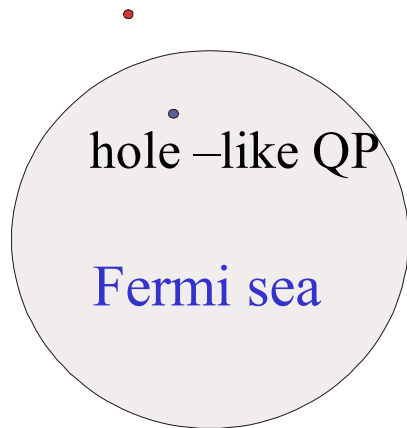
*E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.W. Ramakrishnan,
Phys. Rev. Lett. 42, 673 (1979)*



Fermi liquid and quasiparticles

Quasiparticle concept (Landau 1956, 1957)

electron-like QP



hole-like QP

Fermi sea

Fermi liquid - a system of interacting Fermi particles

Quasiparticle (QP) – excitation in a Fermi liquid, it resembles an excitation in an ideal Fermi gas, but not equivalent

Due to interaction with other electrons and ions, quasiparticle **effective mass** m^* differs from the free electron mass m_e

In superconductors **effective charge** e^* also differs from electronic charge e

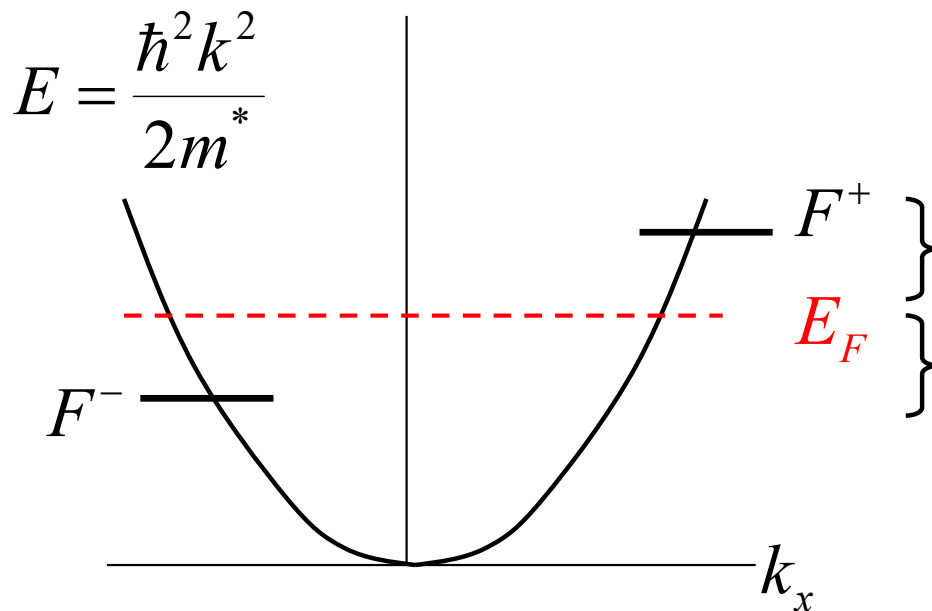
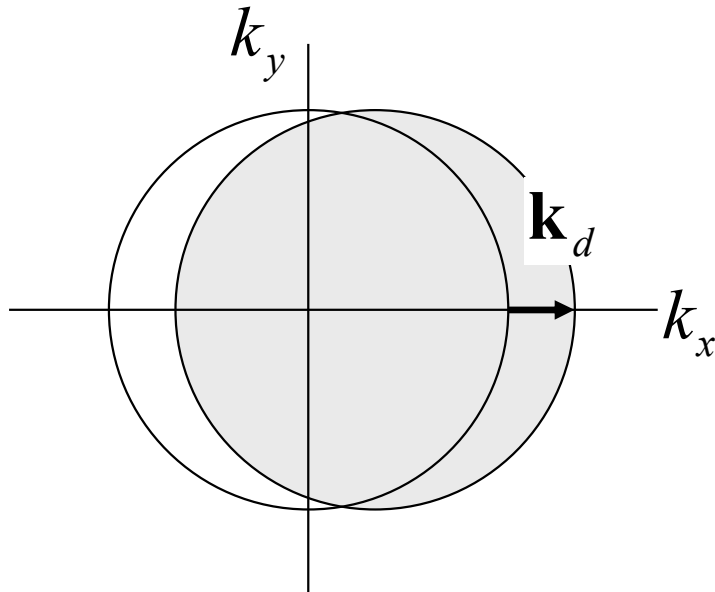
Excitation energy $\xi = p^2 / 2m^* - p_F^2 / 2m^* \approx v_F (p - p_F)$

Quasiparticles have finite **lifetime** due to interaction with other electrons, phonons, etc.

Semi-classical electron transport (Drude-Sommerfeld)

with electric field:
all electrons acquire drift velocity

$$\frac{\hbar \mathbf{k}_d}{m^*} = \mathbf{v}_d = \mu \mathbf{E}$$



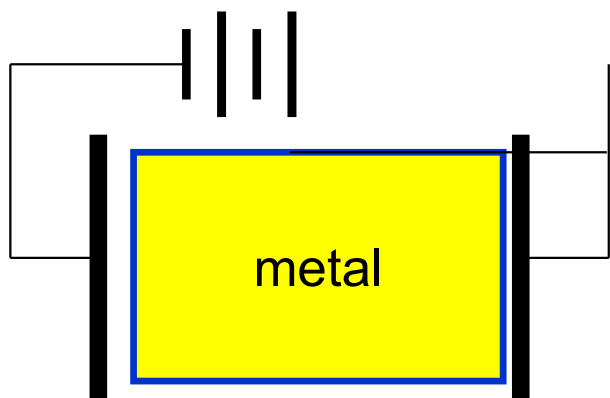
energy diagram:
states occupied with

$$k_x > 0 \quad \text{and} \quad E_F < E \leq F^+$$

states unoccupied with

$$k_x < 0 \quad \text{and} \quad F^- < E \leq E_F$$

Einstein relation for electric conductivity σ : conductance as a diffusion problem



$$n = n(\mu)$$

Density of electrons

$$\frac{dn}{dx} = \frac{dn}{d\mu} \frac{d\mu}{dx} = eE \frac{dn}{d\mu}$$

Chemical potential

Electric field

No current



$$eD \frac{dn}{dx} = \sigma E$$

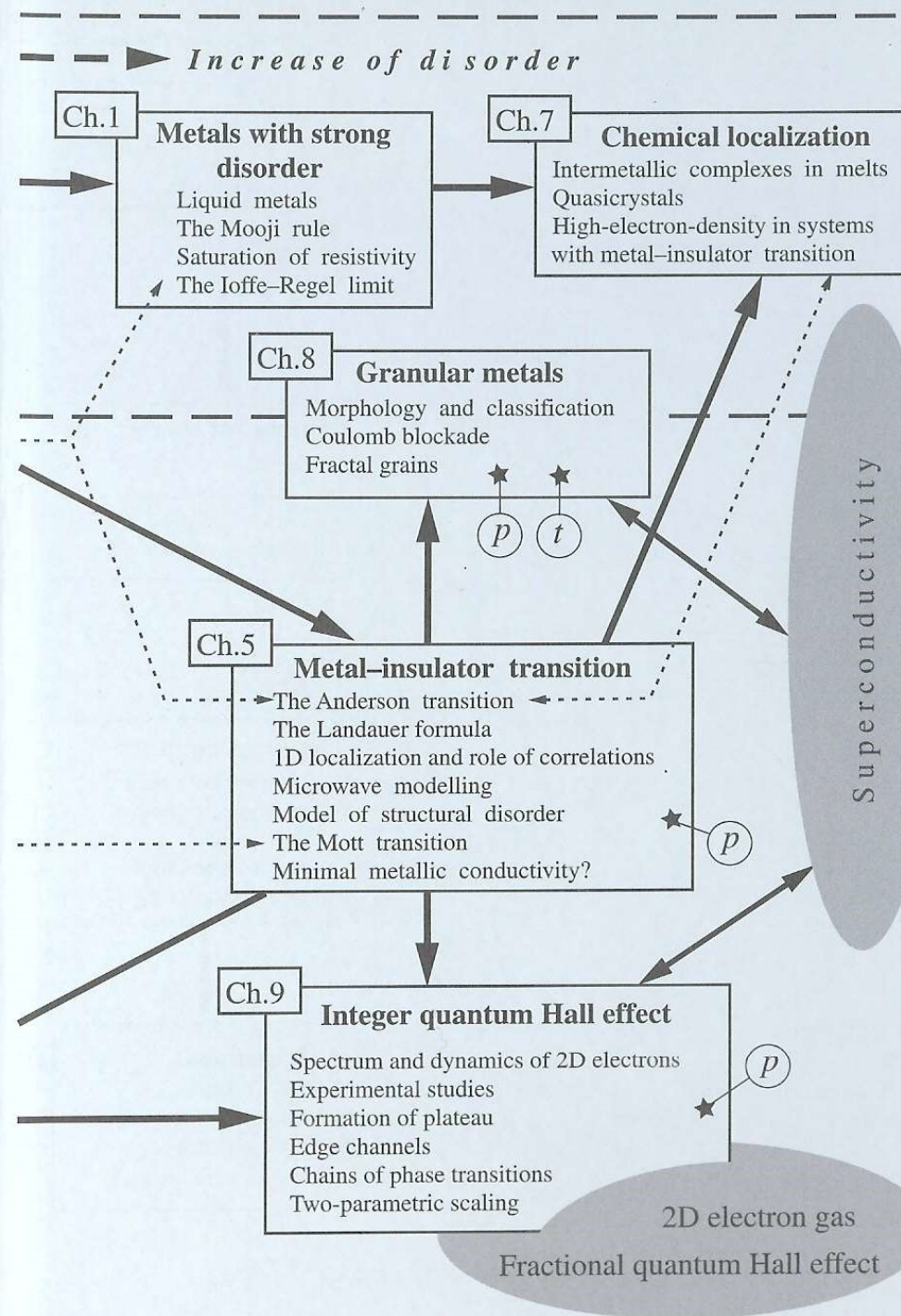
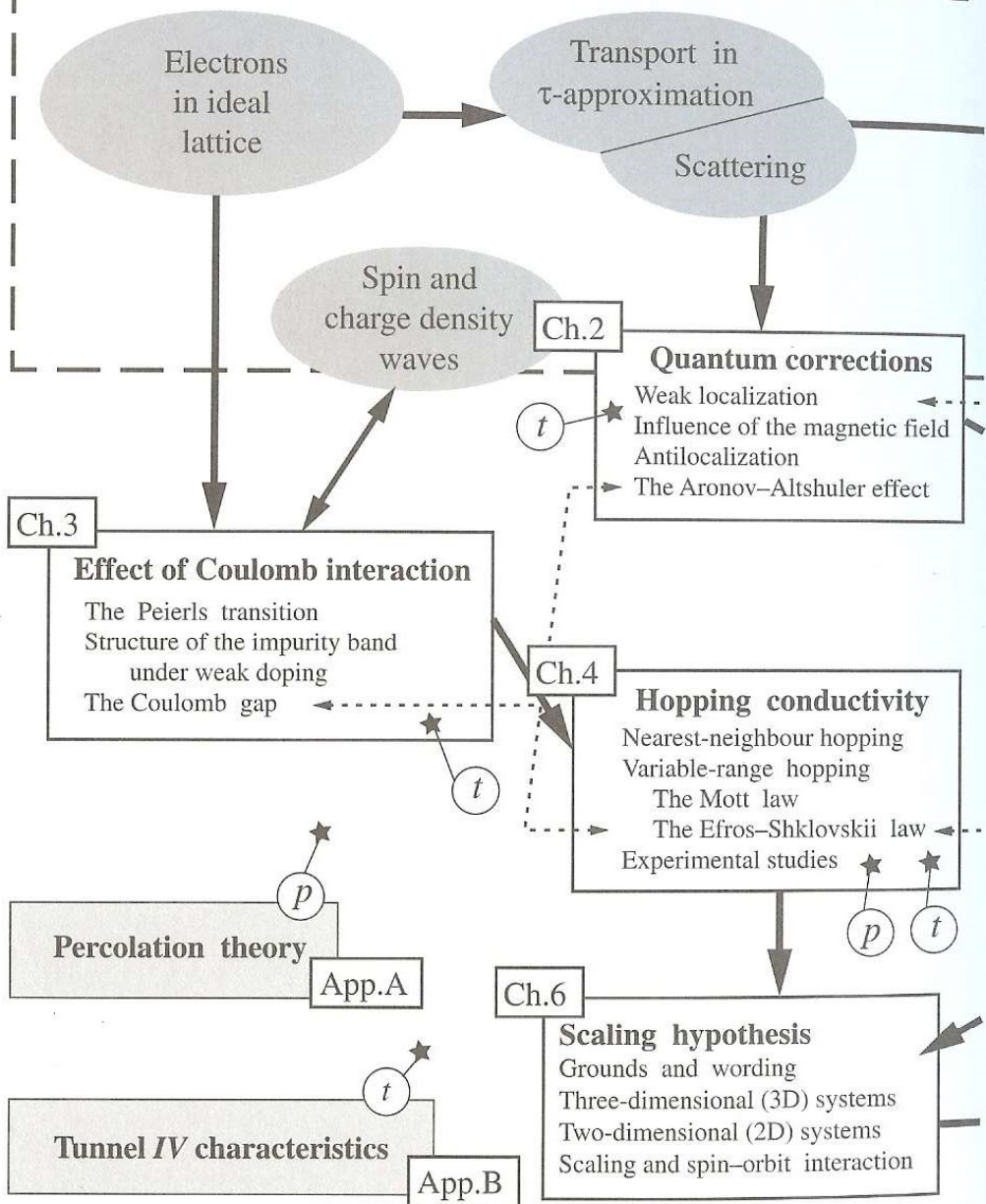
Conductivity

$$\sigma = e^2 D \nu \quad \nu \equiv \frac{dn}{d\mu}$$

Density of states

Diffusion coefficient $D = v l / 3$

High-density electron gas

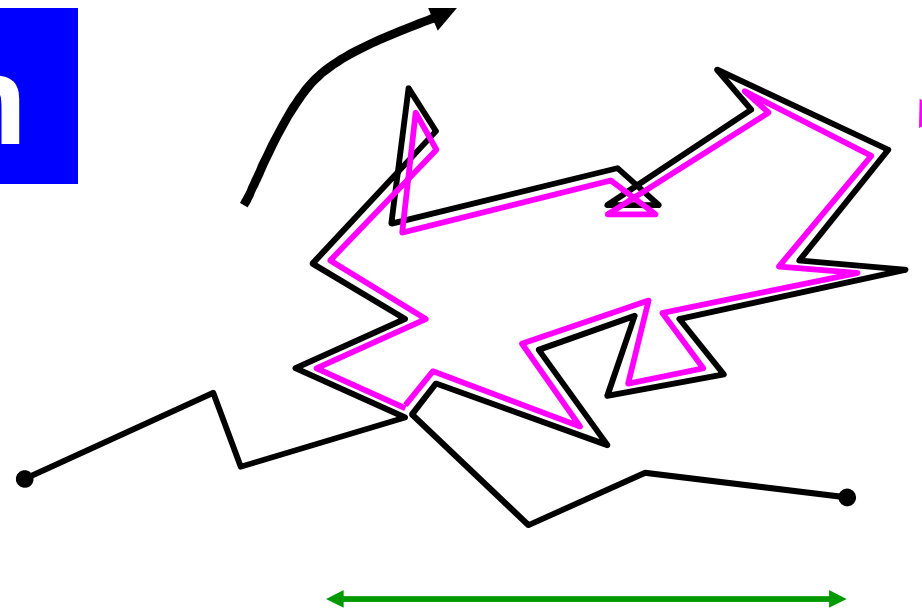


Electrons and disorder in solids
(Schematic guide)

Weak Localization

Quantum corrections to conductivity

$$\sigma = \frac{ne^2 l}{\hbar k_F}$$



Two types of electronic scattering

Elastic scattering, probability $1/\tau$

Inelastic scattering, probability $1/\tau_\phi$

Phase φ of the wave function

$$\tau \ll \tau_\phi$$

- phase coherence

$$\psi \propto \exp(i\varepsilon t / \hbar)$$

$$\varphi = \varepsilon t / \hbar \quad \Rightarrow$$

$$\delta\varphi = \delta\varepsilon t / \hbar$$

Semiclassical description of electric conductivity

At low temperatures conductivity saturates and has the value

$$\sigma = \frac{ne^2l}{\hbar k_F}$$

$k_F l > 1$ – Ioffe-Regel rule

$$k_F = (3\pi^2 n)^{1/3}$$

$$\sigma = A \frac{e^2}{\hbar} n^{1/3} (k_F l)$$

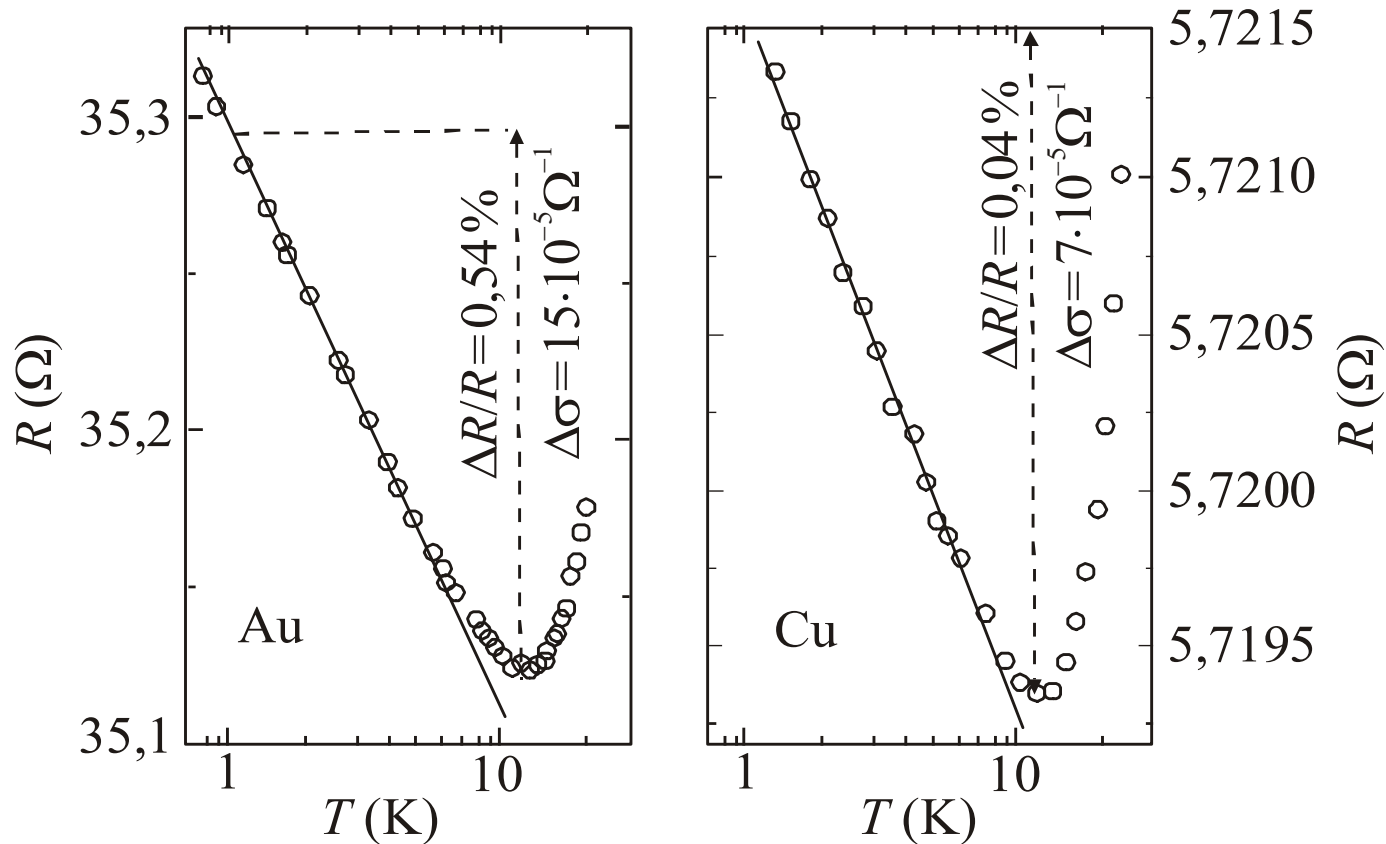
$$n^{-1/3} \approx 3 \text{ \AA}$$

$$\rho_{\max} \sim (100 \div 1000) \mu\Omega \cdot \text{cm}$$

Semiclassical approach should break down for small values of l

First experiments in 1981-1982:

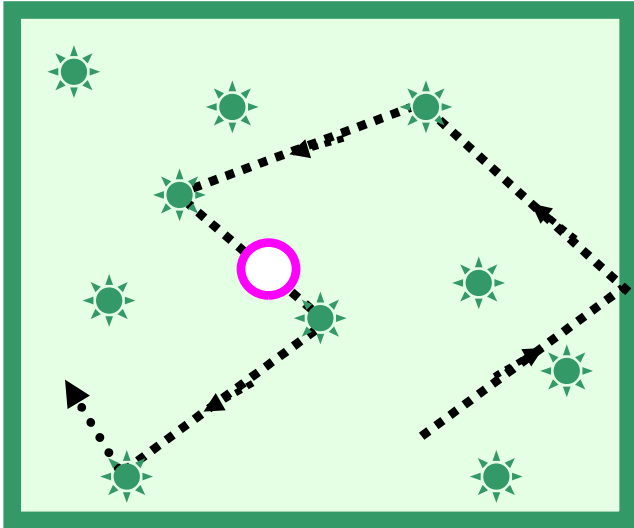
Anomalous behaviour of resistivity of disordered metallic films –
no saturation at low T



Au – S. Dorozhkin, *et al.*,
JETP Lett. **36**, 15 (1982)

Cu – van der Dreis *et al.*, PRL **46**, 565 (1981)

Classical diffusion



Random walk

Density fluctuations $\rho(r, t)$ at a given point in space r and time t .

$$\frac{\partial \rho}{\partial t} - D \nabla^2 \rho = 0 \quad \text{Diffusion Equation}$$

D - Diffusion constant

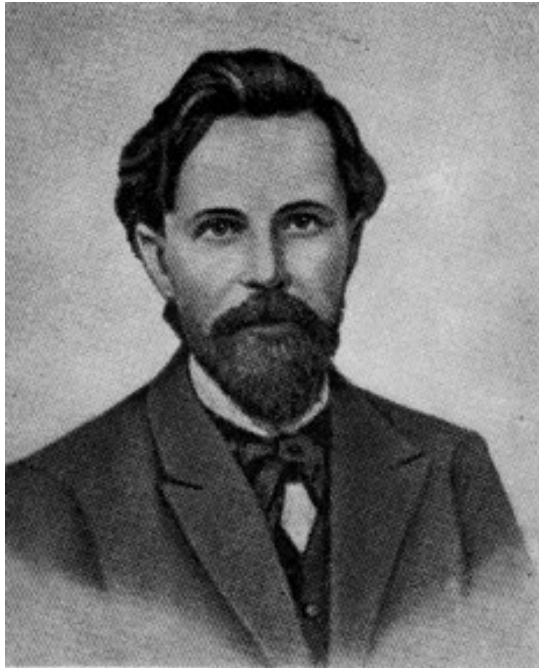
Mean squared distance from the original point at time t

$$\langle r(t)^2 \rangle = Dt$$

Probability to come back (to the element of the volume dV centered at the original point)

$$P(r(t) = 0) dV = \frac{dV}{(Dt)^{d/2}}$$

Markov Chains



Andrei Markov
1856-1922

- A. A. Markov. « Rasprostranenie zakona bol'shih chisel na velichiny, zavisyaschie drug ot druga ». *Izvestiya Fiziko-matematicheskogo obschestva pri Kazanskom universitete*, 2-ya seriya, tom 15, pp 135-156, **1906**.
- A. A. Markov. « **Extension of the limit theorems of probability theory to a sum of variables connected in a chain** ». reprinted in Appendix B of: R. Howard. *Dynamic Probabilistic Systems, volume 1: Markov Chains*. John Wiley and Sons, 1971.

Diffusion description fails at **short** scales

Why?

Einstein: there is no diffusion at too **short** scales - there is memory, i.e., the process is **not marcovian**.

$$r(t) = \sqrt{Dt}$$

$$\frac{dr}{dt} = \sqrt{\frac{D}{2t}}$$

Does velocity diverge at $t \rightarrow 0$?

No because at times shorter than mean free time process is not marcovian and there is no diffusion

Quantum coherence:
there is memory at large distances

Diffusion description fails at **large** scales.
Why ?

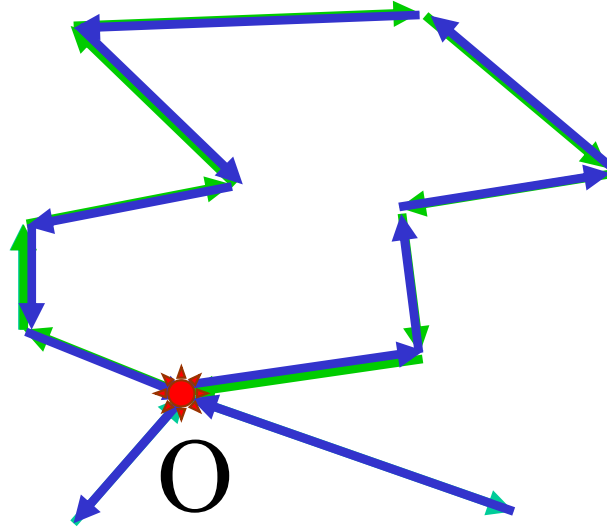
There is phase memory at large distances
in quantum case

Quantum corrections at large conductance -
weak localization

WEAK LOCALIZATION

$$\varphi = \oint \vec{p} d\vec{r}$$

Phase accumulated
when traveling
along the loop



The particle
can go around
the loop in
two directions

$$\varphi_1 = \varphi_2$$

Memory!

Weak Localization

without interference

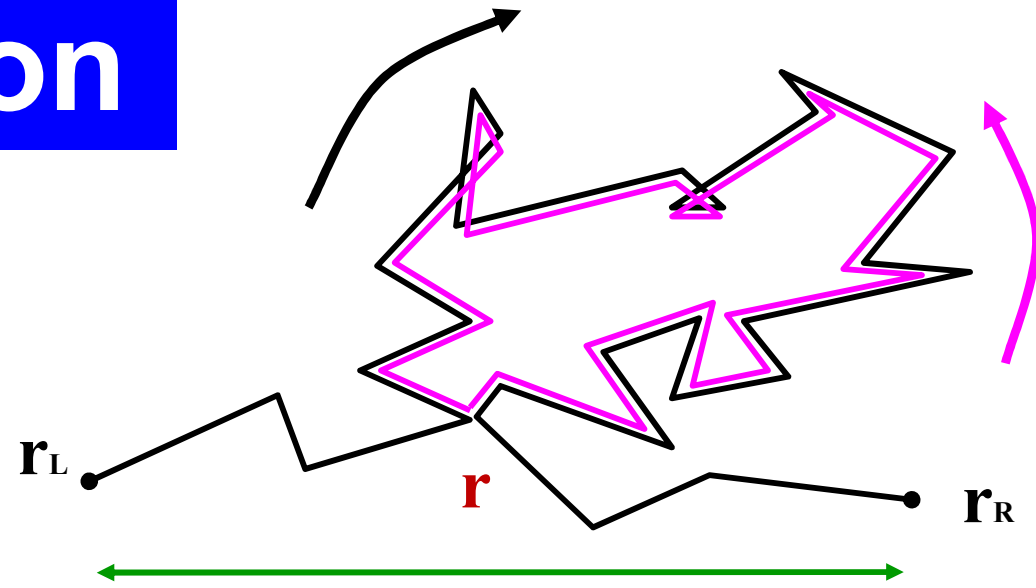
$$|A_1|^2 + |A_2|^2 = 2A^2$$

$$|A_1| = |A_2| = A$$

$\tau \ll \tau_\phi$ - phase coherence

with interference

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2|A_1 A_2| = 4A^2 \quad \rightarrow$$



$A_{1,2}$ are the quantum mechanical amplitudes to return to the point \mathbf{r} by clockwise/counter-clockwise propagation with equal phases $\varphi_1 = \varphi_2$

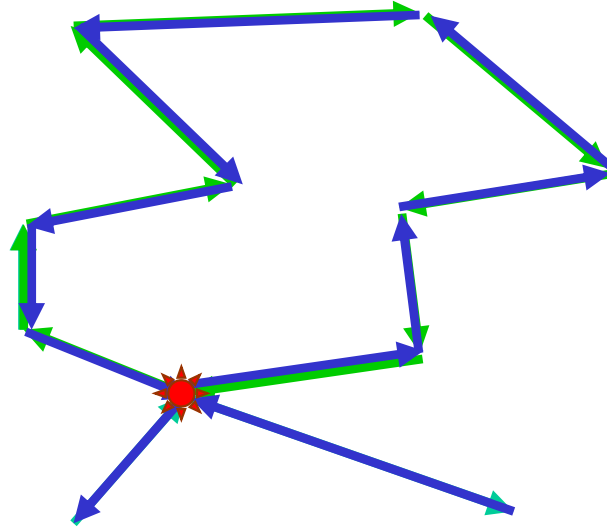
Conductance is defined by the probability of transmission from \mathbf{r}_L to \mathbf{r}_R (left to right)

Probability to return to the point \mathbf{r} increases \Rightarrow Conductance is reduced

WEAK LOCALIZATION

$$\varphi = \oint \vec{p} d\vec{r}$$

Phase accumulated
when traveling
along the loop



The particle
can go around
the loop in
two directions

$$\varphi_1 = \varphi_2$$

Constructive interference \implies *probability to return to the origin gets enhanced* \implies *diffusion constant gets reduced.*

Tendency towards localization

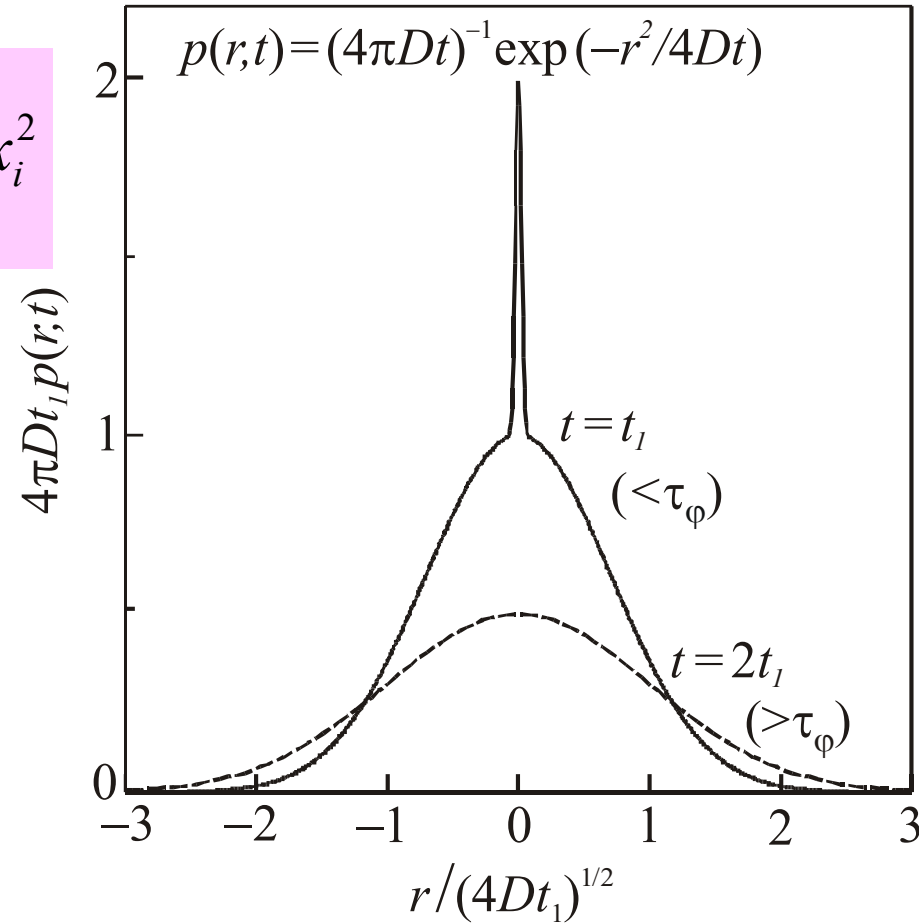
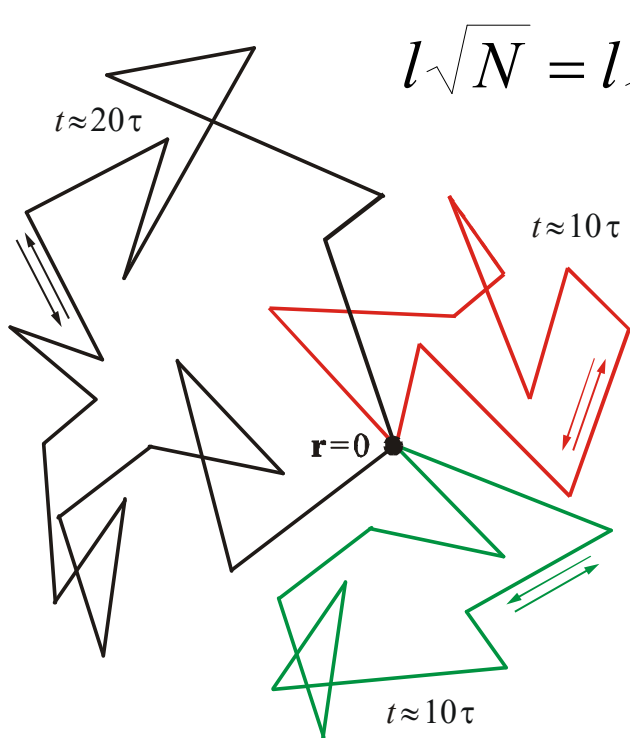
Breakdown of classical diffusion

$$p(r,t) = \frac{1}{(4\pi Dt)^{d/2}} \exp\left(-\frac{r^2}{4Dt}\right), \quad r^2 = \sum_{i=1}^d x_i^2$$

Diffusion coefficient $D = \frac{1}{d} l v \approx \tau v^2 = \frac{l^2}{\tau}$

Distribution width after N steps

$$l\sqrt{N} = l\sqrt{\frac{t}{\tau}} = \sqrt{Dt} = L_N$$



without interference $|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 = 2\mathbf{A}^2$

with interference $|\mathbf{A}_1 + \mathbf{A}_2|^2 =$
 $= |\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2|\mathbf{A}_1 \mathbf{A}_2| = 4\mathbf{A}^2$

How to estimate the correction to the conductivity ?

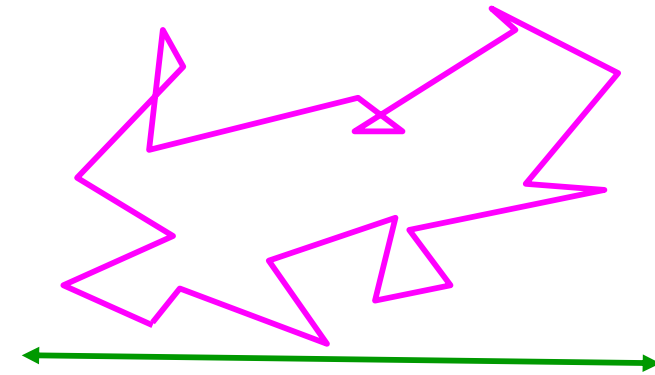
First, we introduce the concept of dimensionality:

Consider a film with thickness b

and compare the phase-breaking length L_ϕ with b

$b < L_\phi \Rightarrow$ dimensionality $d = 2$

$b > L_\phi \Rightarrow$ dimensionality $d = 3$



Typical size of a loop providing the quantum correction is L_ϕ

In 3D correction to the conductivity is proportional to the probability P to come back to the volume element dV :

$$P = dV / (Dt)^{3/2}$$

Since semiclassical trajectory can be viewed as “wire” with diameter of the order of the de Broglie wavelength λ , one can estimate $dV \sim v_F \lambda^2 dt$

$$\text{For } d = 3 \quad \frac{\Delta\sigma}{\sigma} \approx - \int_{\tau}^{\tau_\phi} \frac{v_F \lambda^2 dt}{(Dt)^{3/2}} \approx - \frac{v_F \lambda^2}{D^{3/2}} (\tau^{-1/2} - \tau_\phi^{-1/2}) \approx \frac{1}{k_F^2 l} \left(\frac{1}{L_\phi} - \frac{1}{l} \right)$$

Summary of main results

$$d=3 \quad \frac{\Delta\sigma}{\sigma} \approx - \int_{\tau}^{\tau_{\phi}} \frac{v_F \lambda^2 dt}{(Dt)^{3/2}} \approx - \frac{v_F \lambda^2}{D^{3/2}} (\tau^{-1/2} - \tau_{\phi}^{-1/2}) \approx \frac{1}{k_F^2 l} \left(\frac{1}{L_{\phi}} - \frac{1}{l} \right)$$

const ↑

L_{ϕ} – phase-breaking length:

$$L_{\phi} \approx \sqrt{D\tau_{\phi}} \approx l\sqrt{N} = l(\tau_{\phi}/\tau)^{1/2}$$

τ_{ϕ} – phase-breaking time

$$d=2 \quad \frac{\Delta\sigma}{\sigma} \approx - \int_{\tau}^{\tau_{\phi}} \frac{v_F \lambda^2 dt}{(Dt)b} \approx - \frac{v_F \lambda^2}{Db} \ln \frac{\tau_{\phi}}{\tau}$$

$$d=1 \quad \frac{\Delta\sigma}{\sigma} \approx - \int_{\tau}^{\tau_{\phi}} \frac{v_F \lambda^2 dt}{(Dt)^{1/2} b^2} \approx - \frac{v_F \lambda^2}{Db^2} (L_{\phi} - l)$$

Let us introduce Conductance

$$\sigma_d = \sigma b^{3-d}$$

$$\Delta\sigma_3 \approx \frac{e^2}{\hbar} \left(\frac{1}{L_\phi} - \frac{1}{l} \right)$$

$$\Delta\sigma_2 \approx -\frac{e^2}{\hbar} \ln \frac{L_\phi}{l}$$

$$\Delta\sigma_1 \approx \frac{e^2}{\hbar} (l - L_\phi)$$

expressions for $\Delta\sigma$ do not contain n and σ

Exact result in 2D: $-g_s g_v \frac{e^2}{4\pi^2 \hbar} \ln \left(1 + \frac{\tau_\phi}{\tau} \right)$ C. W. J. Beenakker and H. van Houten
Solid State Physics, 44, 1-228 (1991)

$g_s = 2$ is *spin degeneracy*, g_v is *valley degeneracy* (relevant for semiconductors)

This review is uploaded to Canvas (see the Modul *Additional reading*)

Summary

$$\sigma = e^2 D \nu \quad \nu \equiv \frac{dn}{d\mu}$$

Diffusion coefficient

Density of states

The origin of weak localization:

correction to the diffusion coefficient due to interference, while density of states remains unchanged

(inter-electron interactions are not taken into account)

Weak Localization

Effect of Magnetic Field

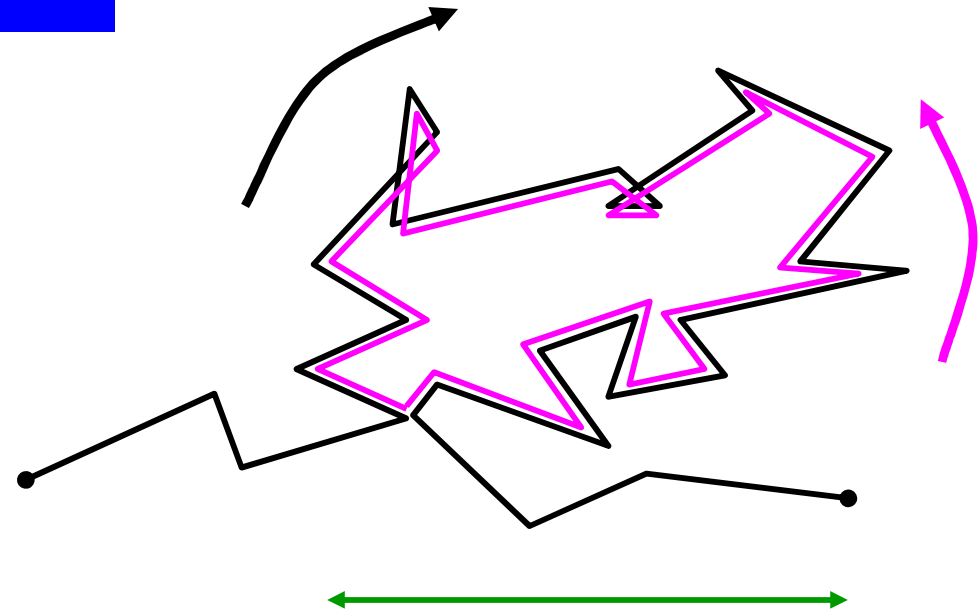
Weak Localization

without interference

$$|A_1|^2 + |A_2|^2 = 2A^2$$

$$|A_1| = |A_2| = A$$

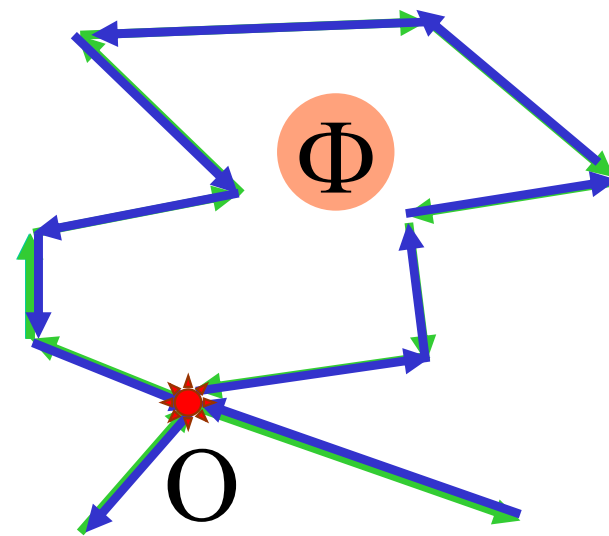
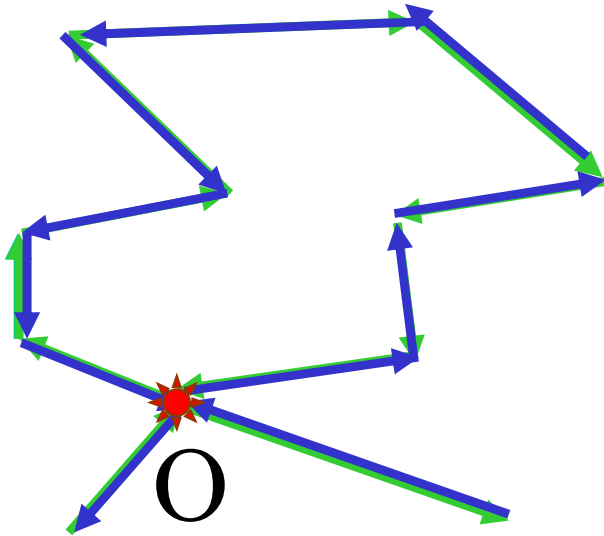
$\tau \ll \tau_\phi$ - phase coherence



with interference

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2|A_1 A_2| = 4A^2$$

Magnetoresistance



No magnetic field

$$\varphi_1 = \varphi_2$$

With magnetic field B

$$\varphi_1 - \varphi_2 = 2\pi \Phi / \Phi_0$$

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2|A_1 A_2| \cos \varphi = 2A^2 (1 + \cos \varphi)$$

$$|A_1| = |A_2|$$

Breaking weak localization by magnetic field

low field regime $\Omega\tau \ll 1$, $\Omega = eB/m$ is the Larmor frequency

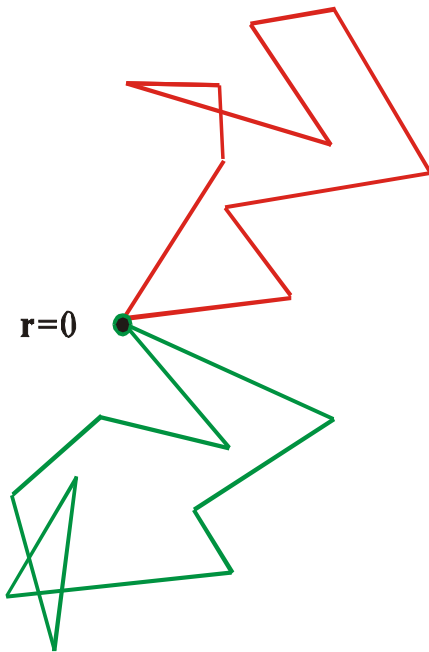
going around trajectory of area S

$$\Psi \rightarrow \Psi \exp\left(i \frac{e}{\hbar} \int \mathbf{A} d\mathbf{l}\right) = \Psi \exp\left(\pm \frac{i\pi BS}{\Phi_0}\right), \quad \Phi_0 = \frac{\pi\hbar}{e} = h/2e$$

Phase difference $\varphi = 2\pi (BS/\Phi_0)$

all diffusive trajectories have different areas S

\Rightarrow weak localization is destroyed.



Average area \overline{S} and flux $B\overline{S}$ depend on time

$$\overline{BS} \approx B\overline{r^2} \approx BDt$$

How to estimate the “breaking magnetic field”

Phase difference $\varphi = 2\pi (BS/\Phi_0)$

We use $\overline{BS} \approx Br^{-2} \approx BDt$ and replace t by τ_φ

Since $D\tau_\varphi = L_\varphi^2$

one can formulate the condition of weak localization breaking

$$\varphi = 2\pi (BL_\varphi^2/\Phi_0) \simeq 1$$

Breaking field

$$B_\varphi = \frac{\Phi_0}{\pi L_\varphi^2} = \frac{\hbar}{e} (D\tau_\varphi)^{-1}$$

Magnetic length and magnetic time

$$l_B = \left(\frac{\hbar}{2eB} \right)^{1/2}$$

$$\tau_B = \frac{l_B^2}{D} \approx \frac{\Phi_0}{BD} \approx \Omega^{-1} (k_F l)^{-1}$$

Breaking weak localization by magnetic field

In strong magnetic field

$$\int_{\tau}^{\tau_{\phi}} \rightarrow \int_{\tau}^{\tau_B}$$

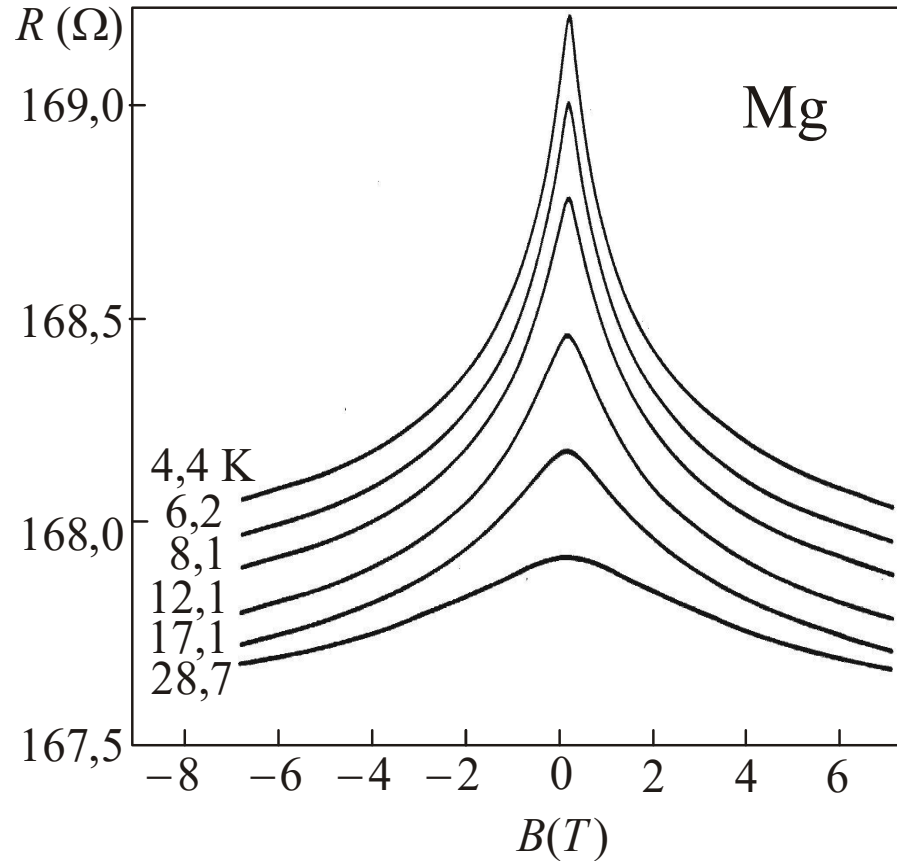
in 2 dimensions

$$\Delta\sigma(B) - \Delta\sigma(0) \sim \frac{e^2}{\hbar} \ln \frac{L_{\phi}}{l_B}$$

$$l \ll l_B \leq L_{\phi}$$

Magnetic length

$$l_B = (\Phi_0 / B)^{1/2}$$



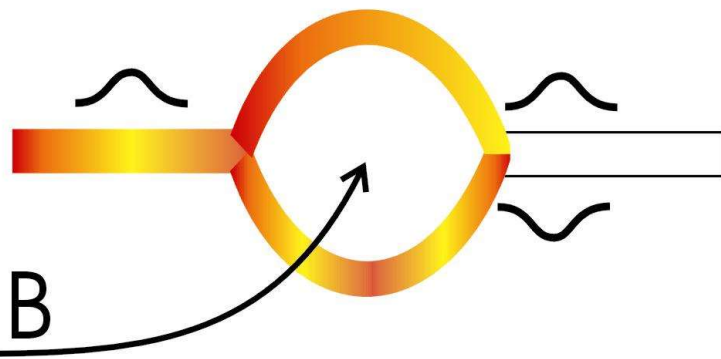
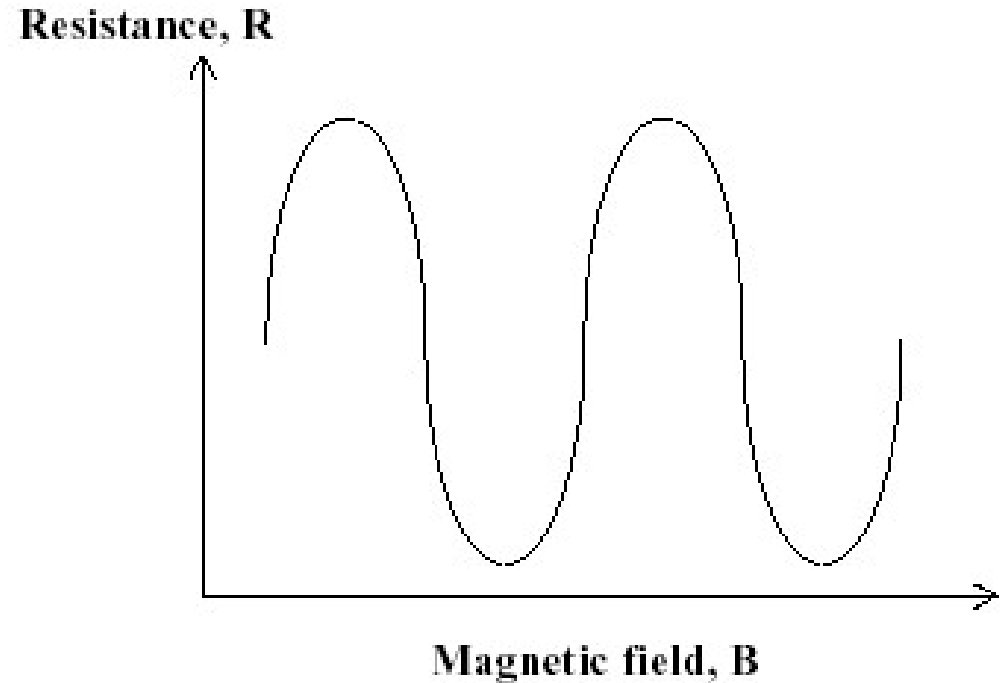
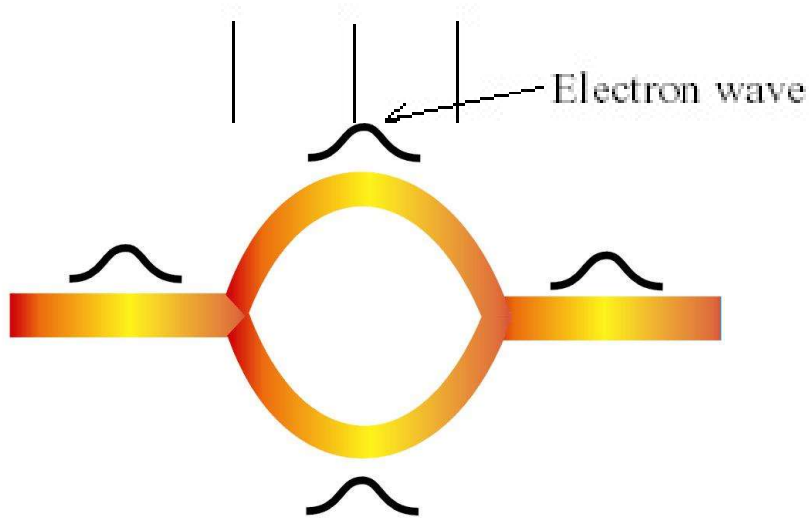
G.Bergmann, Phys.Rep. **107**, 1 (1981)

Two characteristic fields:

$$B_{\phi} \simeq \Phi_0 / L_{\phi}^2$$

$$B_l \simeq \Phi_0 / l^2$$

Aharonov-Bohm interference effects



The particle can go around the loop in two directions

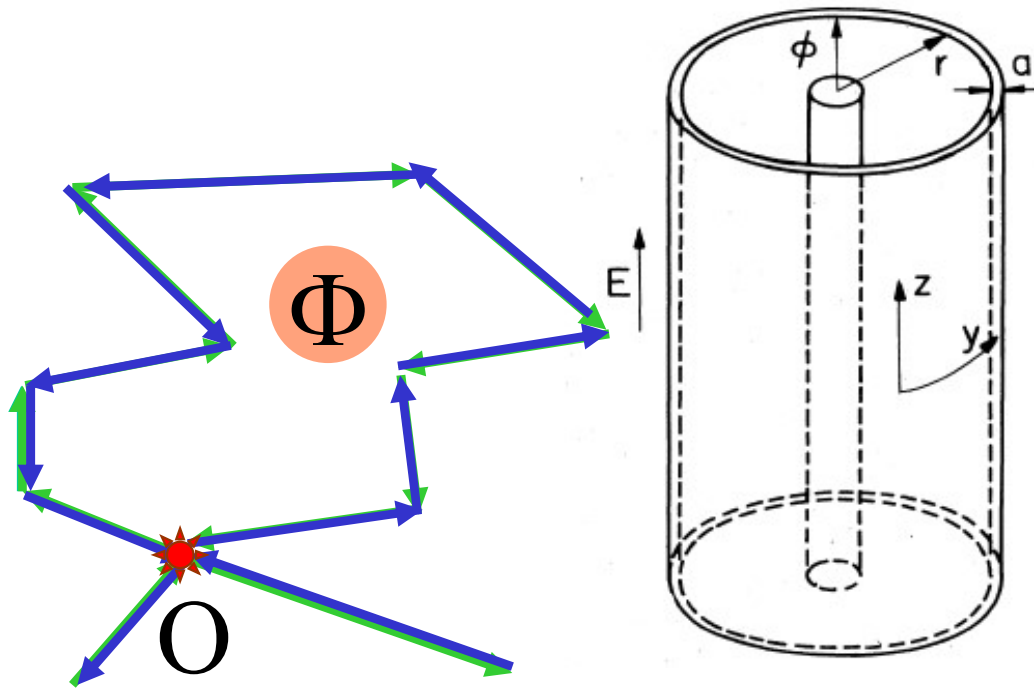
Phase accumulated when traveling along the loop

$$\varphi = \oint (\vec{p} - e\vec{A}) d\vec{r}$$

Resistance is a periodic function of the magnetic flux with the period

$$\Phi_0 = h/e$$

Aharonov-Bohm effect in the WL regime



With magnetic field H

$$\varphi_1 - \varphi_2 = 2\pi \Phi / \Phi_0$$

Resistance is a periodic function of the magnetic flux with the period

$$\Phi_0 = h/2e$$

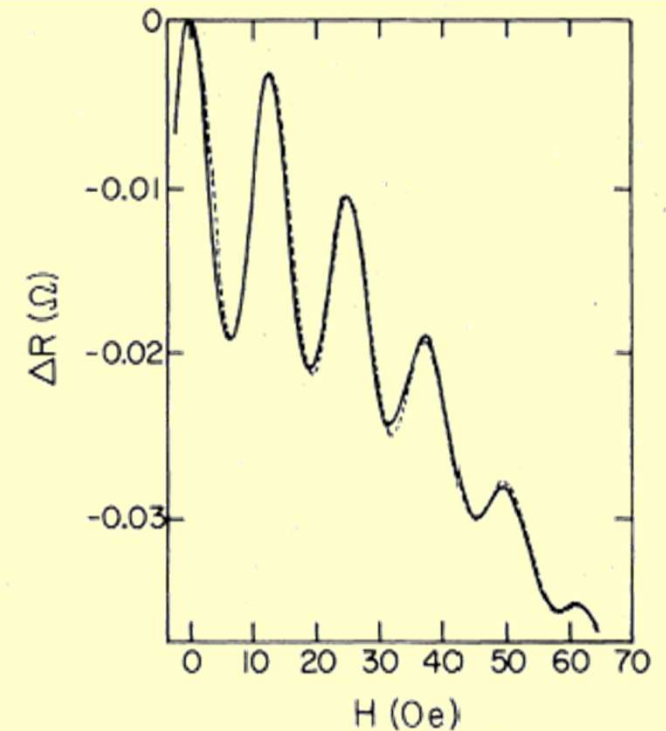
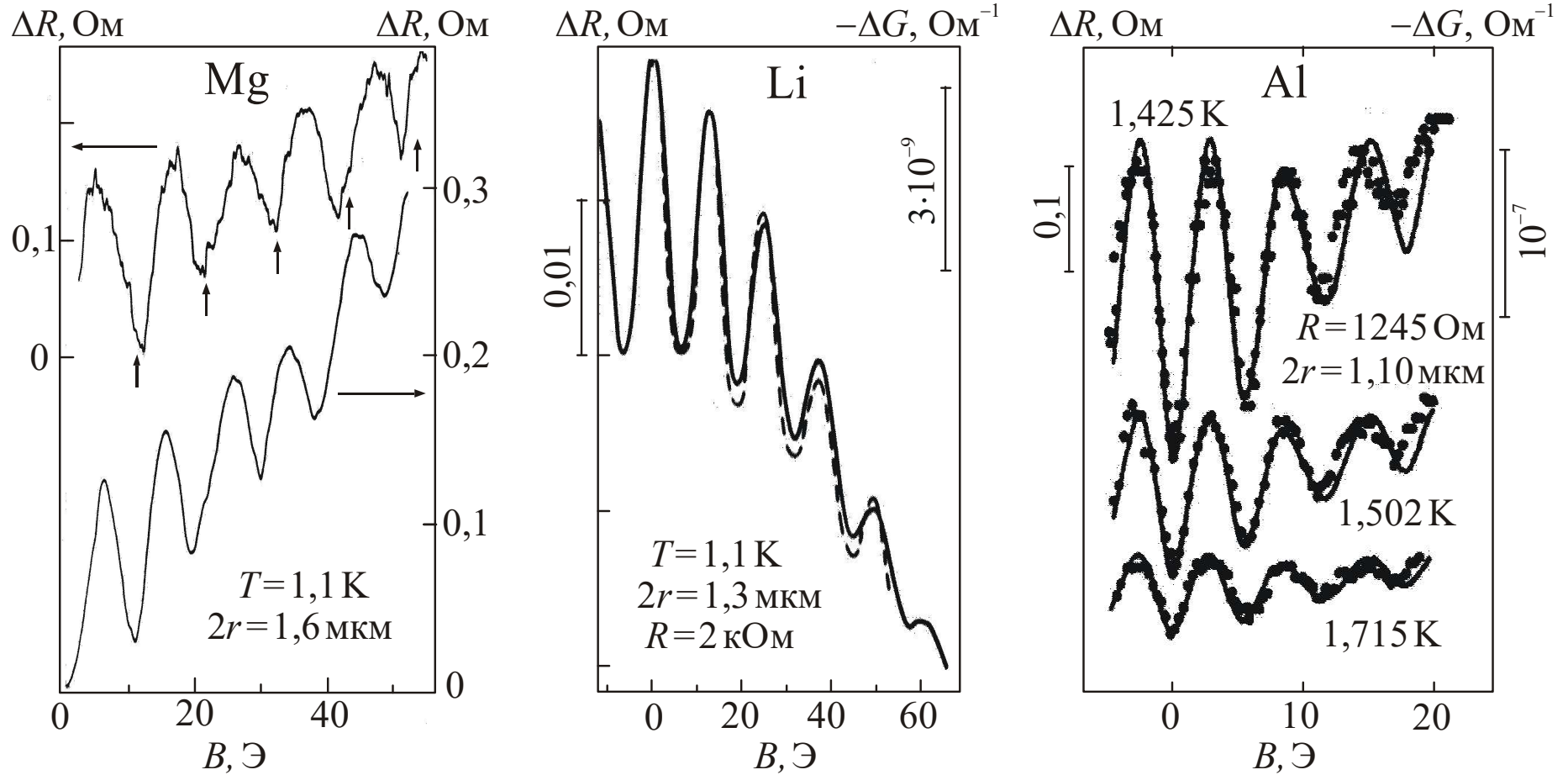
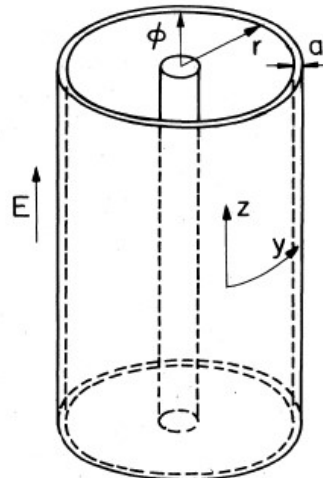


FIG. 8. Longitudinal magnetoresistance $\Delta R(H)$ at $T=1.1$ K for a cylindrical lithium film evaporated onto a 1-cm-long quartz filament. $R_{4,2}=2$ k Ω , $R_{300}/R_{4,2}=2.8$. Solid line: averaged from four experimental curves. Dashed line: calculated for $L_{\varphi}=2.2$ μm , $\tau_{\varphi}/\tau_{s0}=0$, filament diameter $d=1.31$ μm , film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d=1.30\pm 0.03$ μm (Altshuler *et al.*, 1982; Sharvin, 1984).

Magnetoresistance of cylindrical films

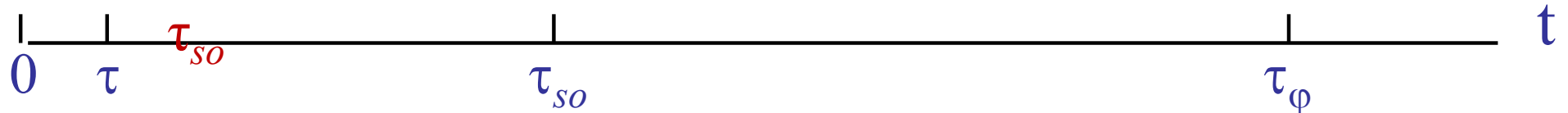


Sharvin et al, JETP Lett.
34, 285 (1981);
35, 476 (1982)



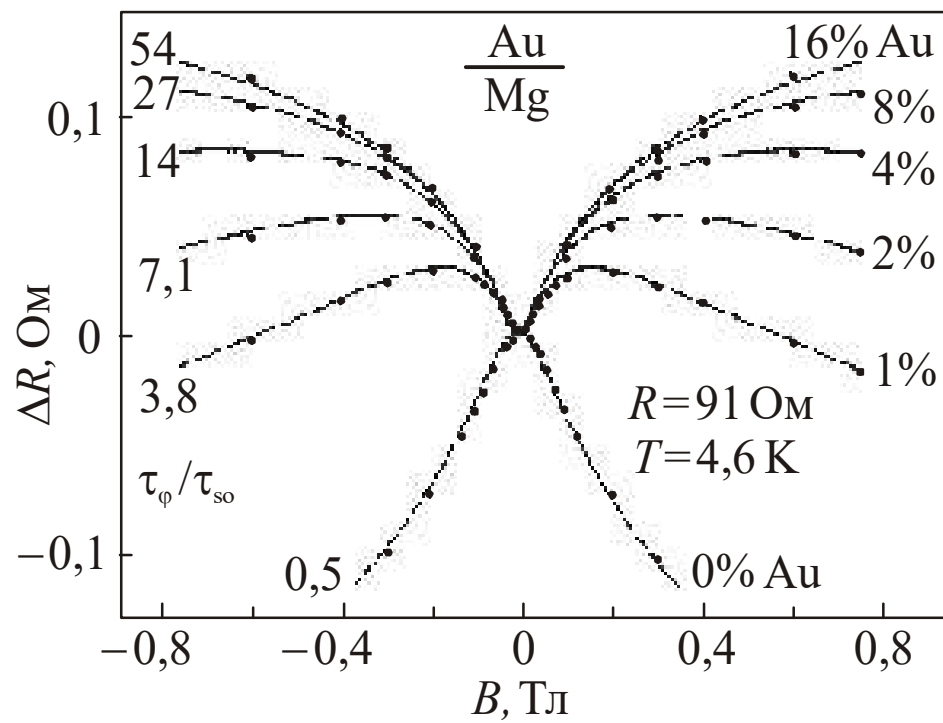
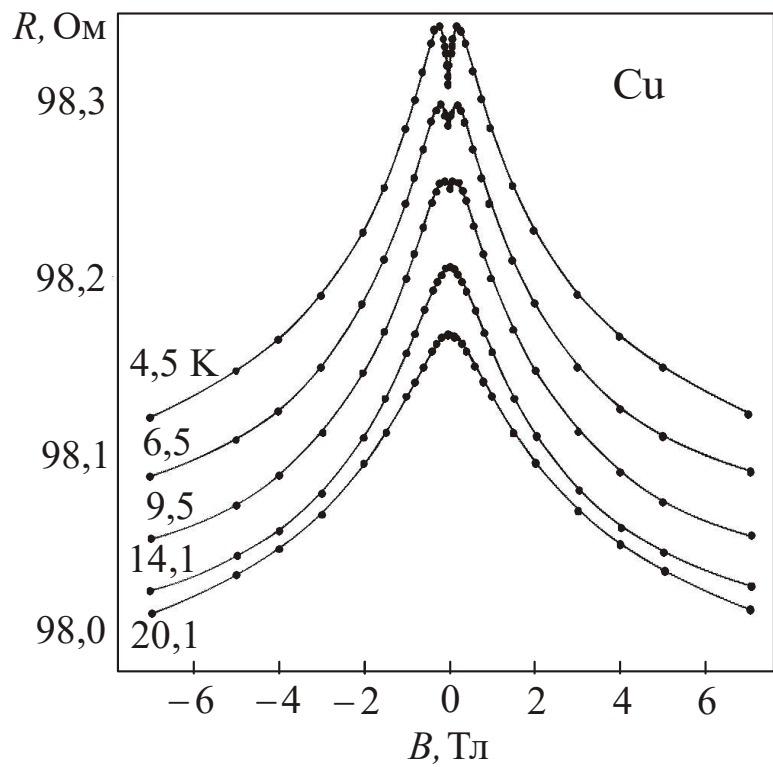
M. Gijs et al.,
Phys.Rev.B 30, 2964
(1984)

Weak antilocalization: spin-orbit coupling time



$$\Psi = \begin{pmatrix} \Psi_0 \\ \Psi_{1,-1} \\ \Psi_{1,0} \\ \Psi_{1,1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} (\varphi_+^{(1)} \varphi_-^{(2)} - \varphi_-^{(1)} \varphi_+^{(2)}) \\ \varphi_-^{(1)} \varphi_-^{(2)} \\ \frac{1}{\sqrt{2}} (\varphi_+^{(1)} \varphi_-^{(2)} + \varphi_-^{(1)} \varphi_+^{(2)}) \\ \varphi_+^{(1)} \varphi_+^{(2)} \end{pmatrix} \quad \begin{array}{l} \text{1 singlet state} \\ \text{3 triplet states} \end{array}$$

$$\frac{\Delta\sigma_d}{\sigma} \approx - \int_{\tau}^{\tau_\phi} \frac{v_F \lambda^2 dt}{(Dt)^{d/2} b^{3-d}} \left(\frac{3}{2} e^{-t/\tau_{so}} - \frac{1}{2} \right), \quad d = 1, 2, 3$$



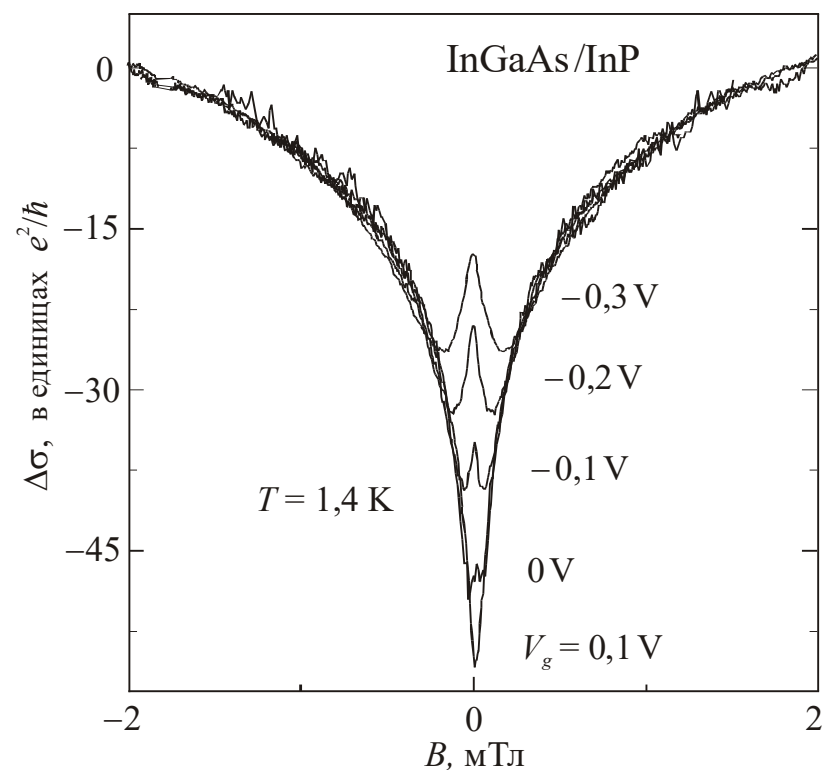
G.Bergmann, Phys.Rep. **107**, 1 (1981)

Heterostructures:

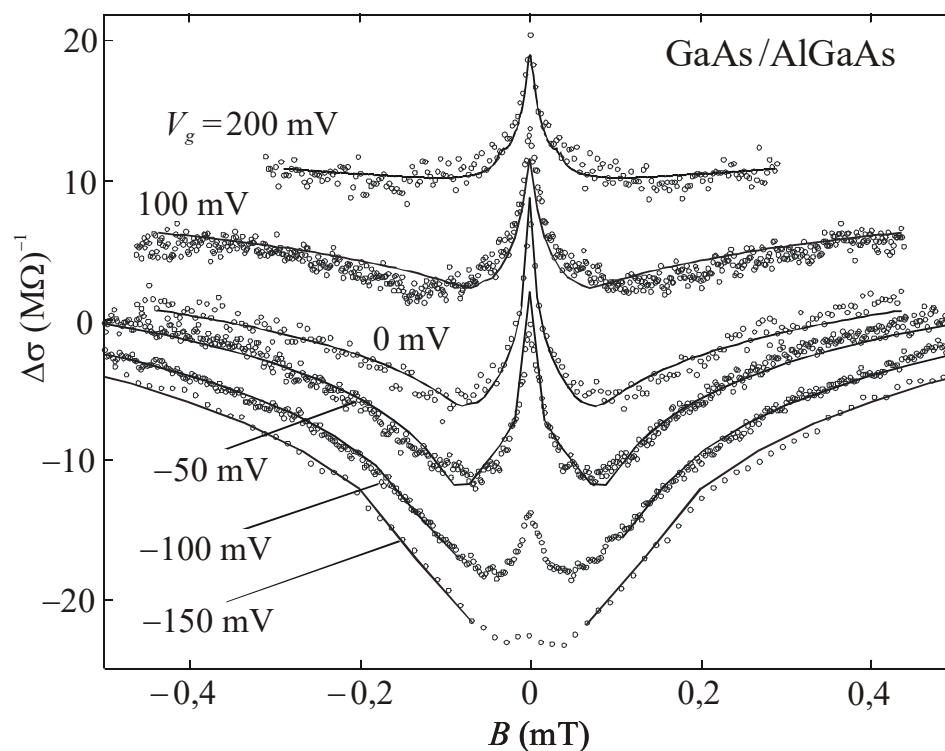
spin-orbit interaction $\sim Ee \propto E [\mu\nu]$

dependence on external field : via ν .

=> Dependence on gate potential

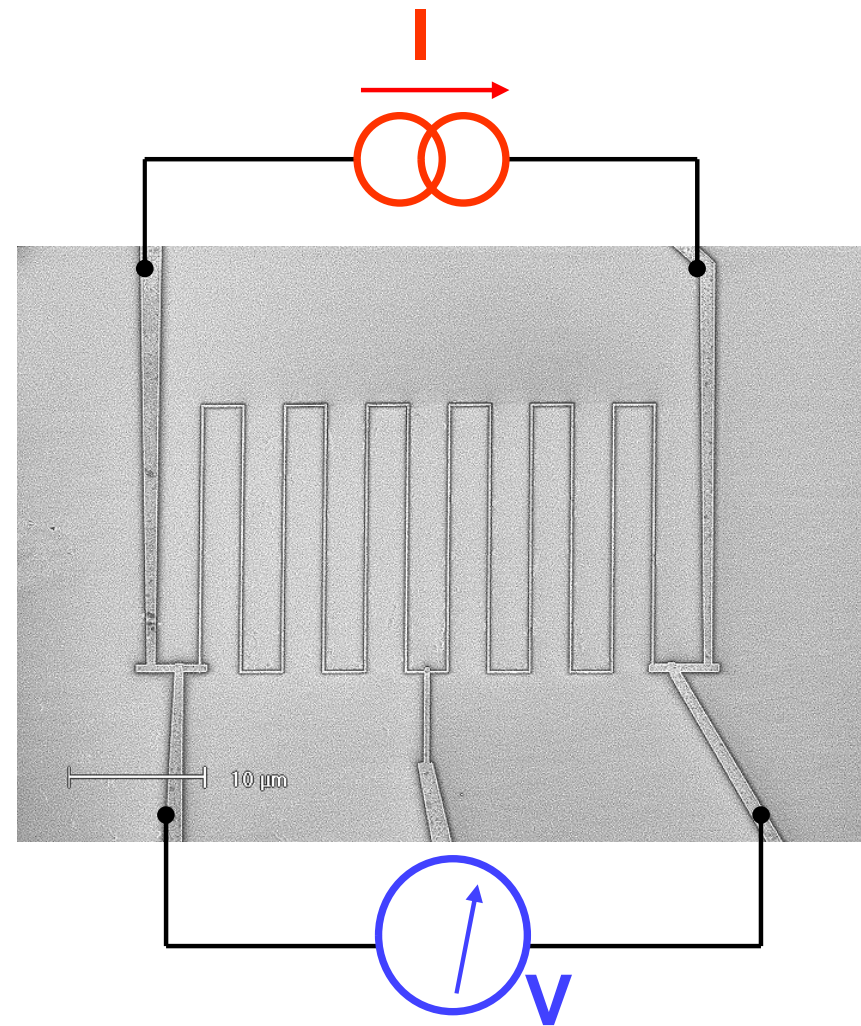


S.A. Studenikin *et al.*,
JETP L:ett. **77**, 362 (2003)

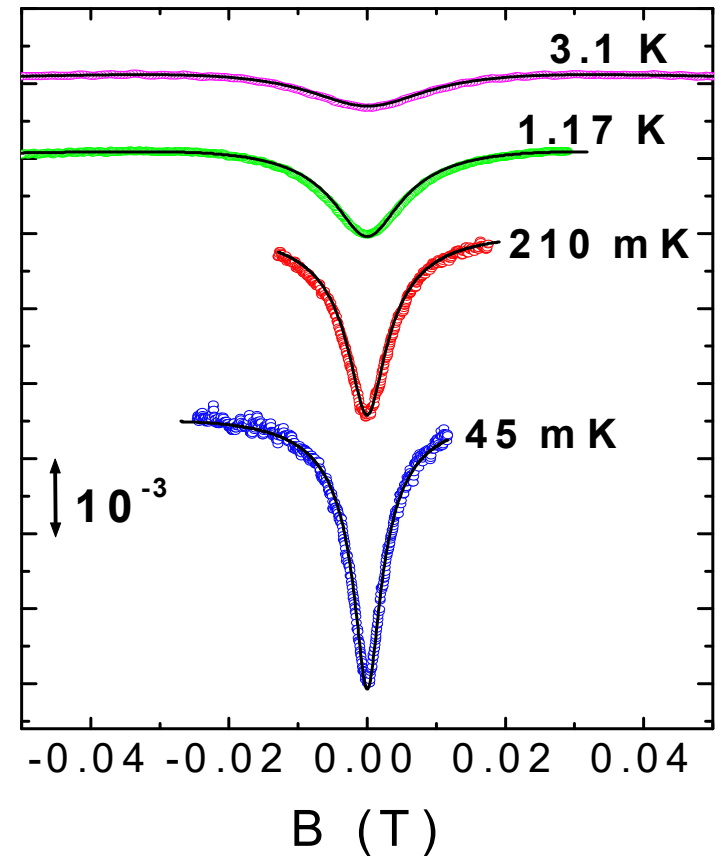


J.B. Miller *et al.*,
PRL **90**, 076807 (2003)

Weak Localization and dephasing rate



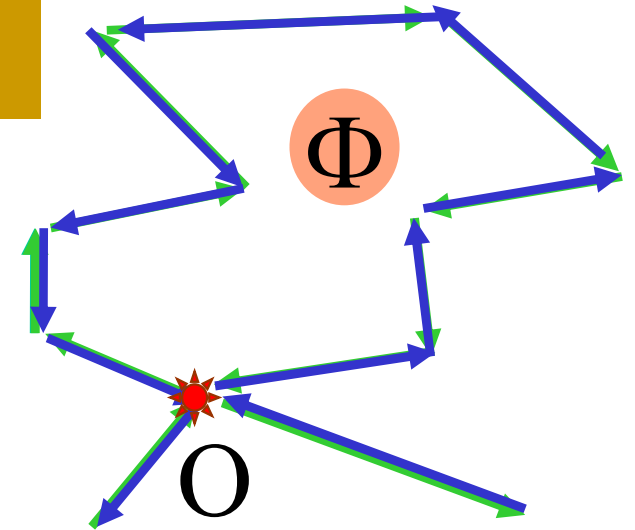
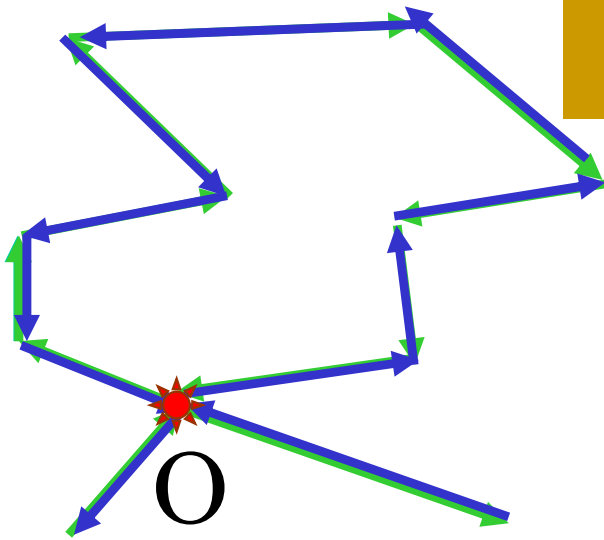
$\Delta R/R$



Echternach, Gershenson, Bozler, Bogdanov & Nilsson,

PRL 48, 11516 (1993)

Magnetoresistance

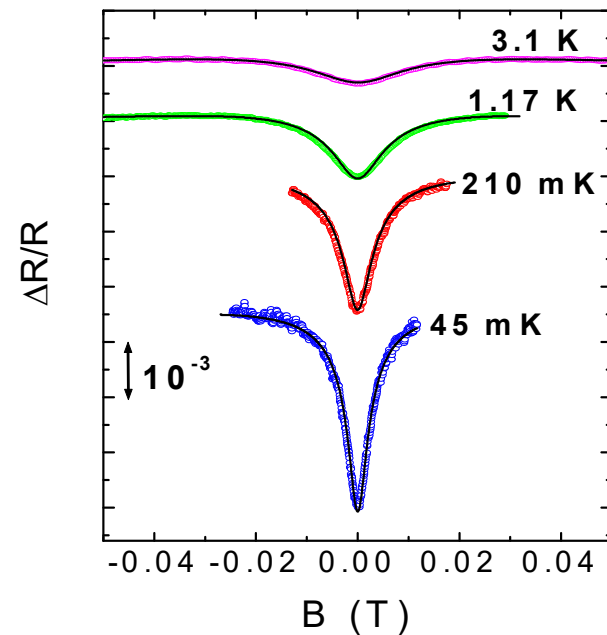
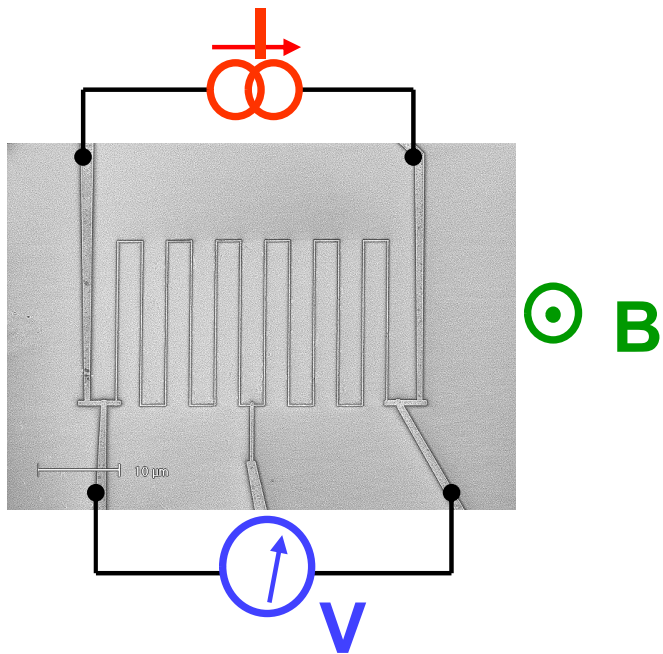


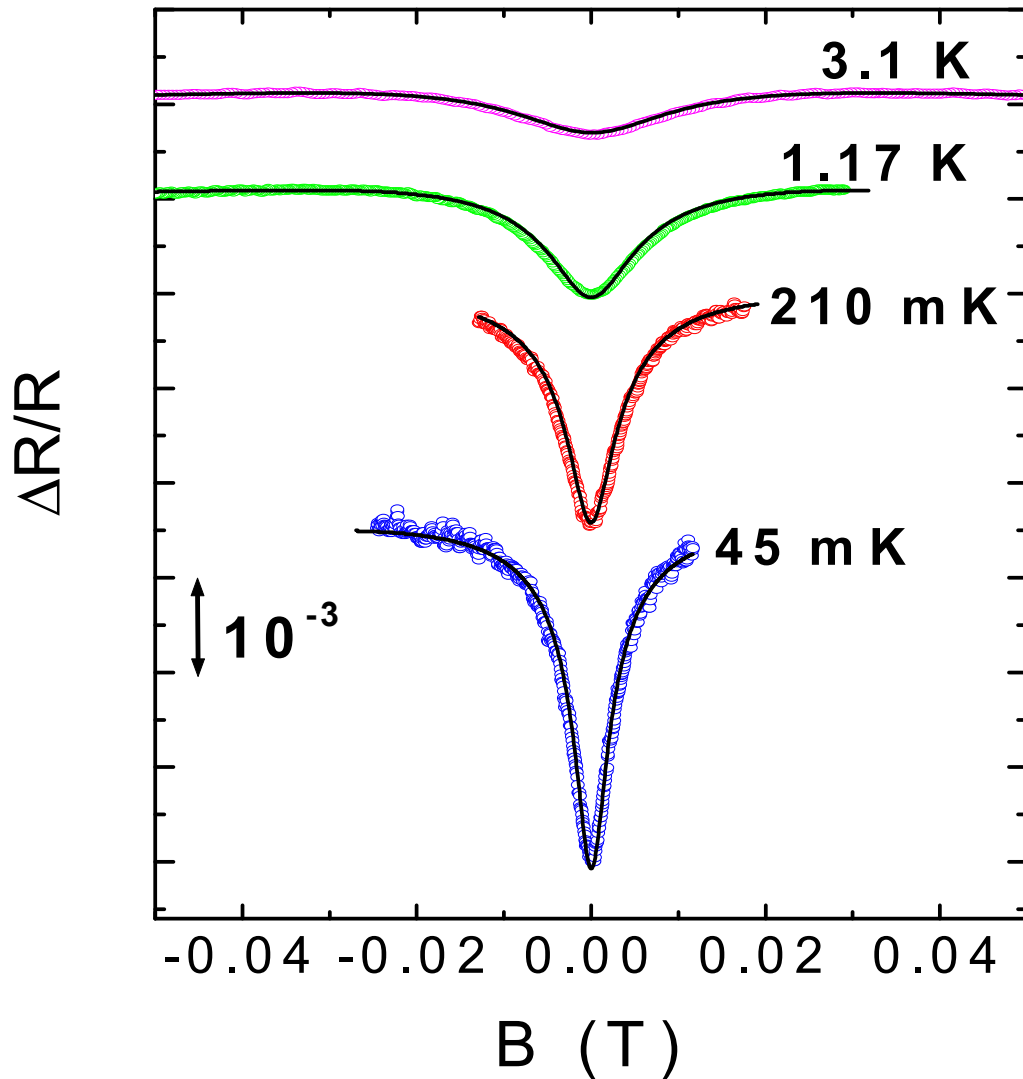
No magnetic field

$$\varphi_1 = \varphi_2$$

With magnetic field B

$$\varphi_1 - \varphi_2 = 2\pi \Phi / \Phi_0$$





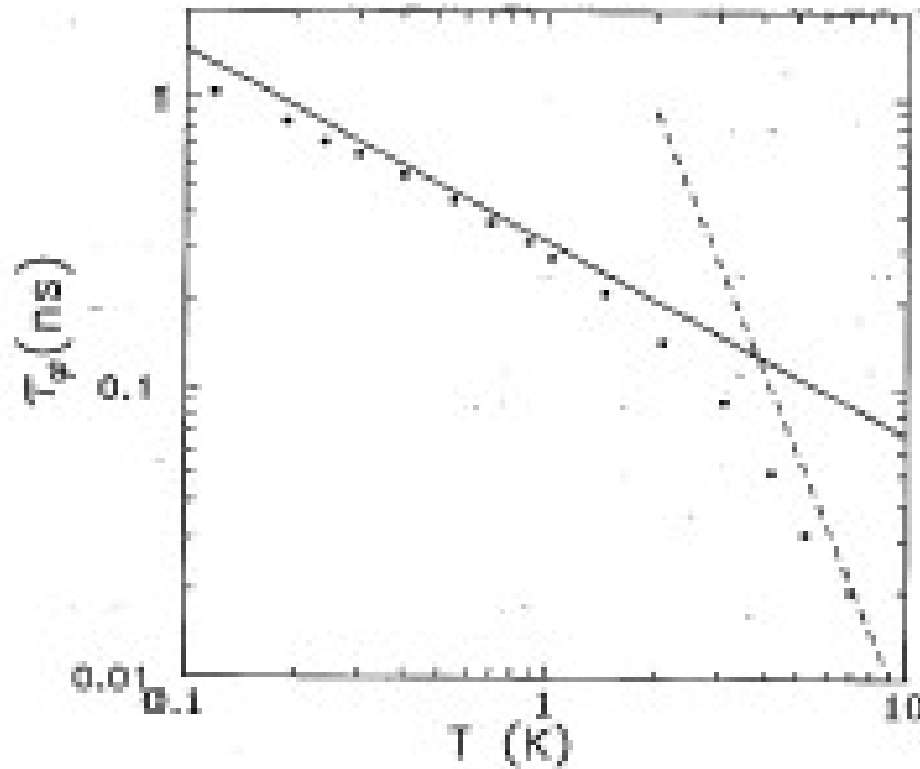
$$\frac{\Delta R}{R} \approx -\frac{\hbar}{e^2 L} R \sqrt{L_\phi^2 + \left(\frac{A}{l_B}\right)^2}$$

$$L_\phi = \sqrt{D\tau_\phi} \quad l_B = \sqrt{\hbar/2eB}$$

L is the length of the wire
 A is the wire cross-section

Dephasing rate can be measured

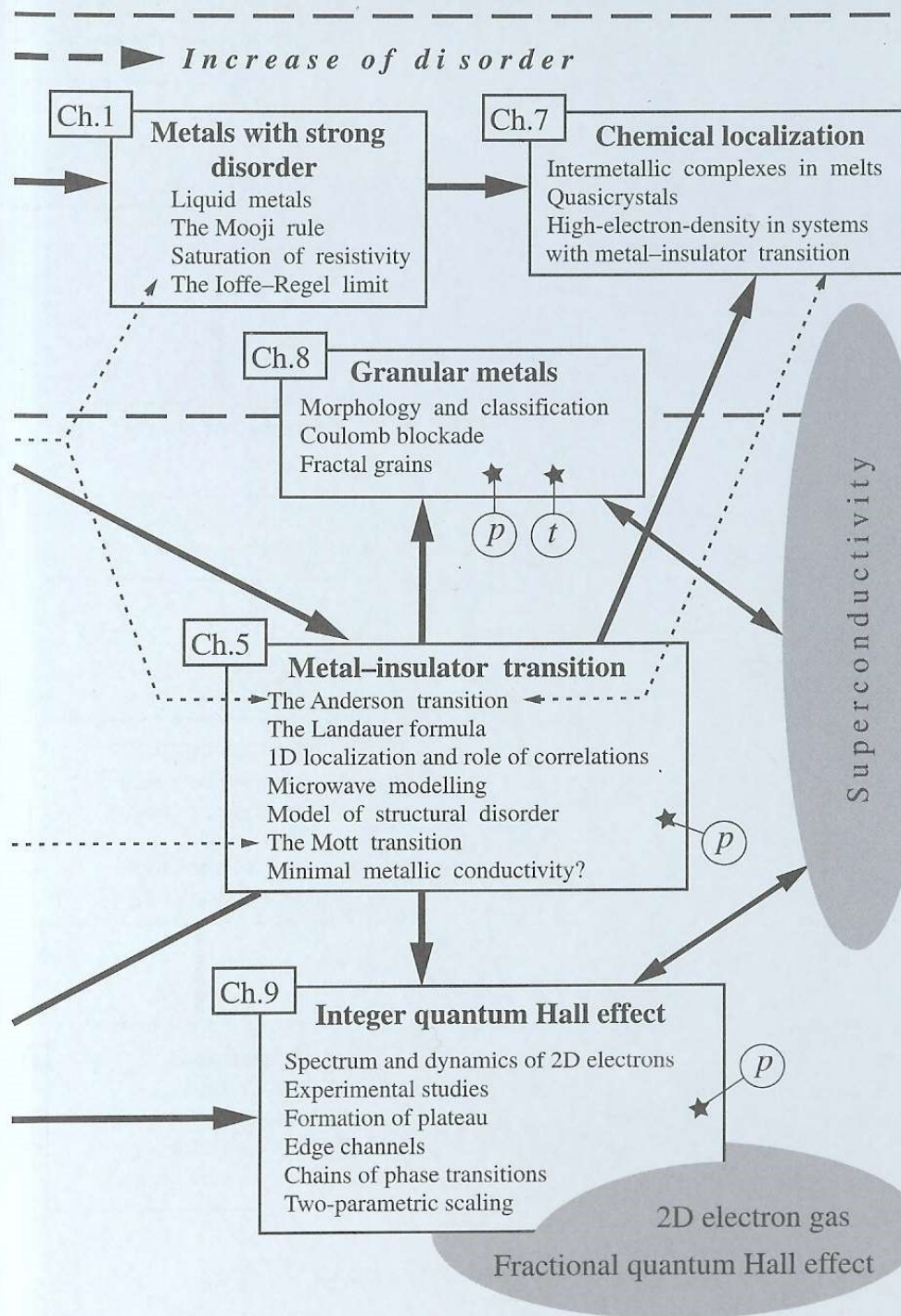
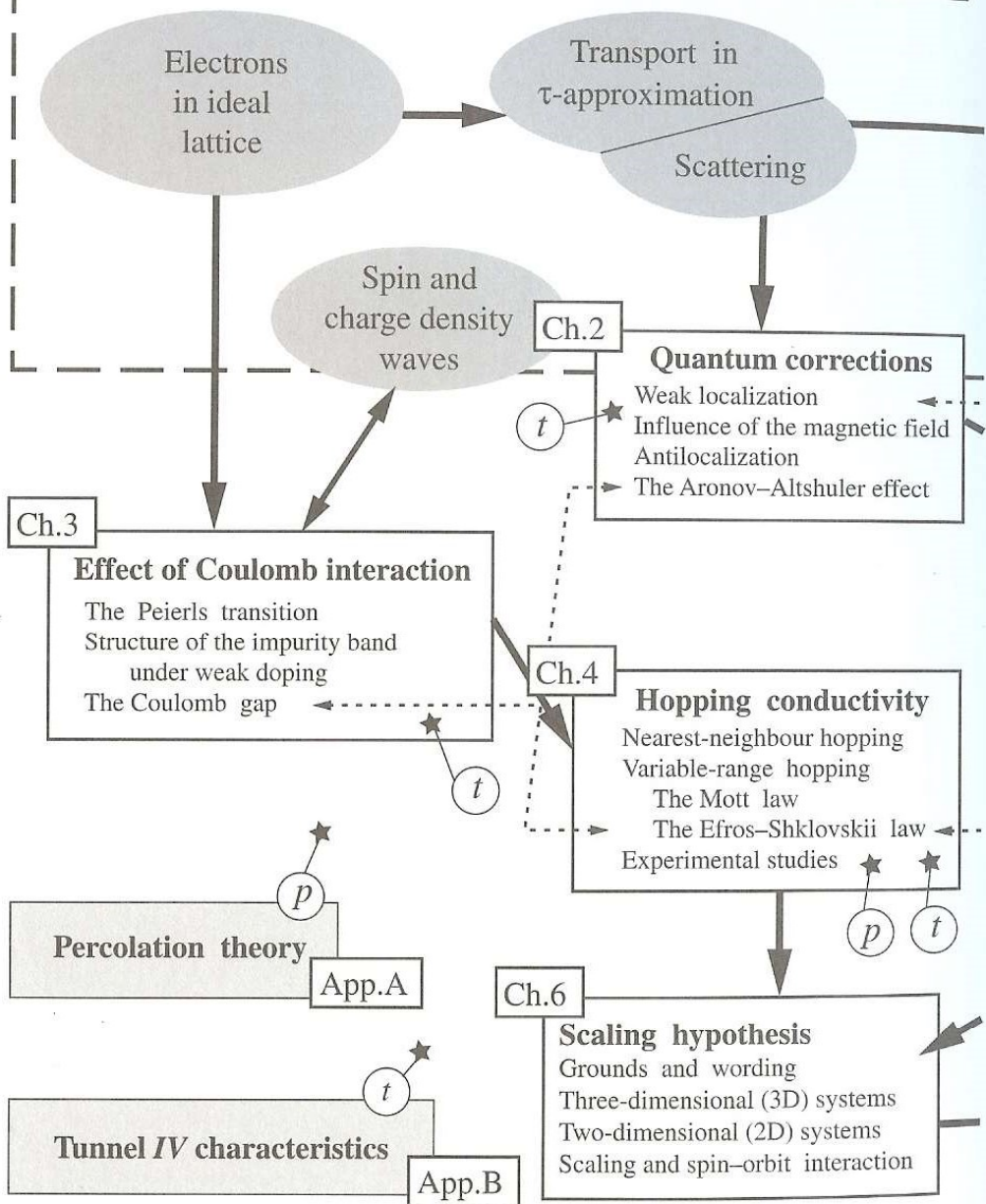
Temperature dependence of τ_φ (from magnetoresistance)



Echternach, Gershenson, Bozler, Bogdanov & Nilsson,
PRB 48, 11516 (1993)

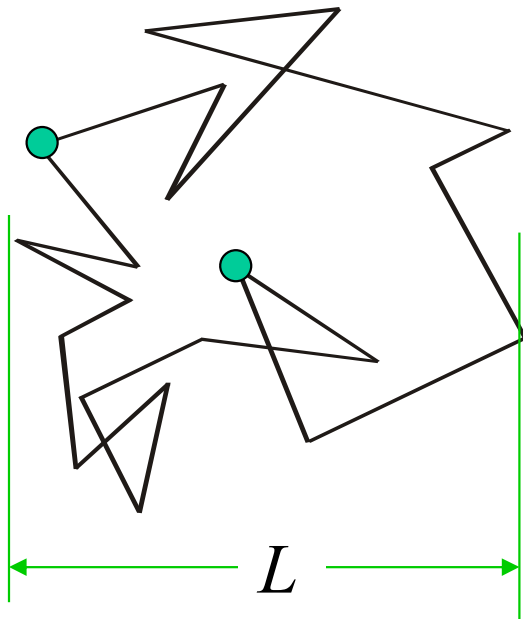
Quantum corrections to
conductance due to e-e
interaction (the density
of states effect)

High-density electron gas



Electrons and disorder in solids
(Schematic guide)

Dephasing due to ee – interaction



ballistic

$$r \sim v_F t$$

diffusive

$$r \sim l \sqrt{t/\tau} \sim v_F \sqrt{t\tau}$$

phase $\exp(i\varphi) = \exp[i(\varepsilon_i/\hbar)t], \quad \Delta\varphi = (\Delta\varepsilon/\hbar)t$

dephasing time

$$\tau_{ee} \sim \hbar/\Delta\varepsilon$$

$$\Delta\varepsilon \sim T$$

$$\tau_{ee} \sim \hbar/T$$

dephasing length

$$L_{ee} \approx \sqrt{D\tau_{ee}} \approx \sqrt{\frac{\hbar D}{T}}$$

Electrons diffuse and keep their coherence during time τ_{ee}

typical distance during this time is L_{ee} .

Thouless energy

Using the expression for the dephasing length

$$L_{ee} \approx \sqrt{D\tau_{ee}} \approx \sqrt{\frac{\hbar D}{T}}$$

and replacing L_{ee} by the system size L

we define the corresponding energy scale, *the Thouless energy*

$$E_T = \hbar D / L^2$$

E_T has a meaning of the *inverse diffusion time* of the traveling through the system.

It determines the “phase coherent” energy interval around Fermi energy for a given system size **L**

Exchange interaction

Consider two electrons in states 1,2 with orbital wave functions $\varphi_1(r)$ and $\varphi_2(r)$

Total wave function
$$\varphi = \frac{1}{\sqrt{2}} [\varphi_1(r_1) \varphi_2(r_2) \pm \varphi_1(r_2) \varphi_2(r_1)]$$

sign '+' for total spin $S = 0$, sign '-' for spin $S = 1$

since the sum of the orbital (L) and spin (S) quantum numbers should be even

Average values of interaction energy $U(r_2 - r_1)$ are $A \pm J$

$$A = \iint U |\varphi_1(r_1)|^2 |\varphi_2(r_2)|^2 dV_1 dV_2,$$
$$J = \iint U \varphi_1(r_1) \varphi_1^*(r_2) \varphi_2(r_2) \varphi_2^*(r_1) dV_1 dV_2$$

\Rightarrow shifts of the energy levels $\Delta E_0 = J, \Delta E_1 = -J$

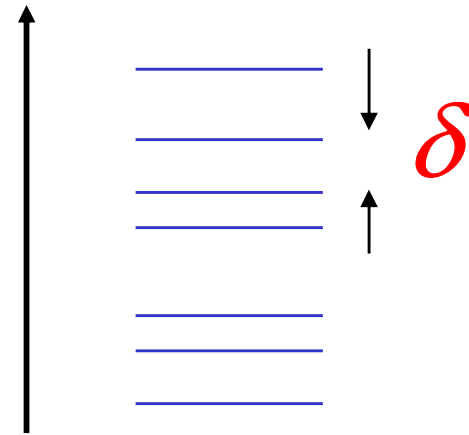
J - exchange energy

Exchange interaction and the density of states

Mean level spacing

$$\delta = 1/gL^d$$

g is the density of states



Exchange interaction leads to the shift
of the energy levels

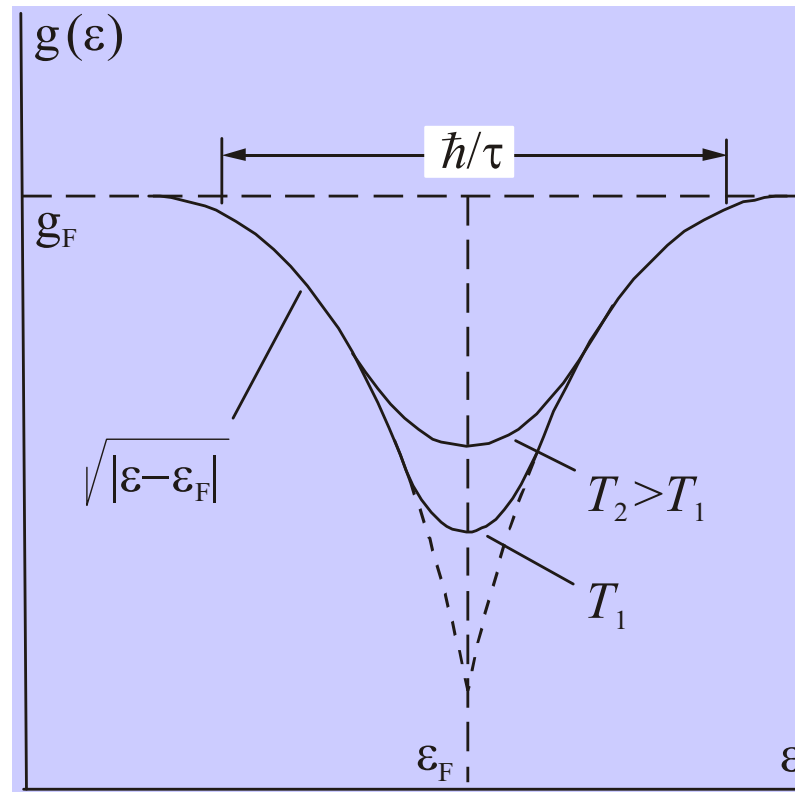
$$\Delta E_0 = J, \quad \Delta E_1 = -J$$

\Rightarrow *Effective “level repulsion” \Rightarrow Reduction of the density of states*

*This effect is realized only in the
“phase coherent” energy range determined by*

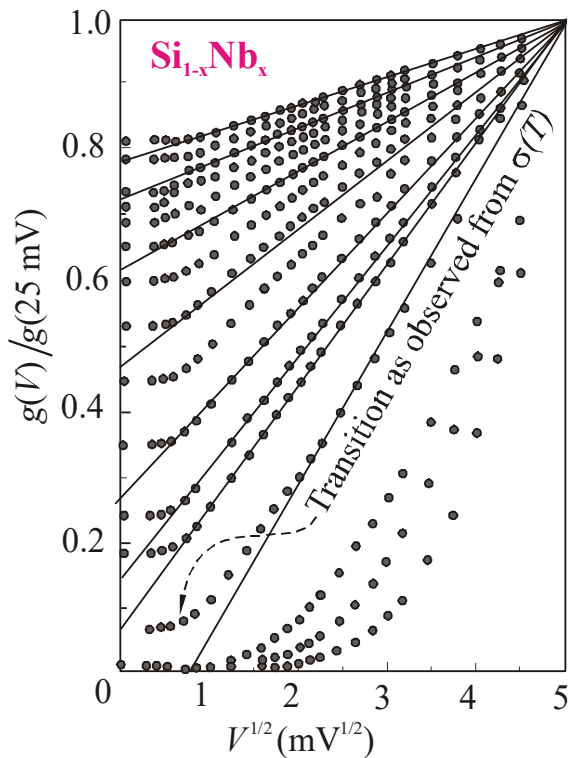
$$\tau_{ee} \sim \hbar / \Delta \varepsilon$$
$$\Delta \varepsilon \sim T$$

*The result of level repulsion -
correction to the density of states:
Altshuler - Aronov (AL) effect*



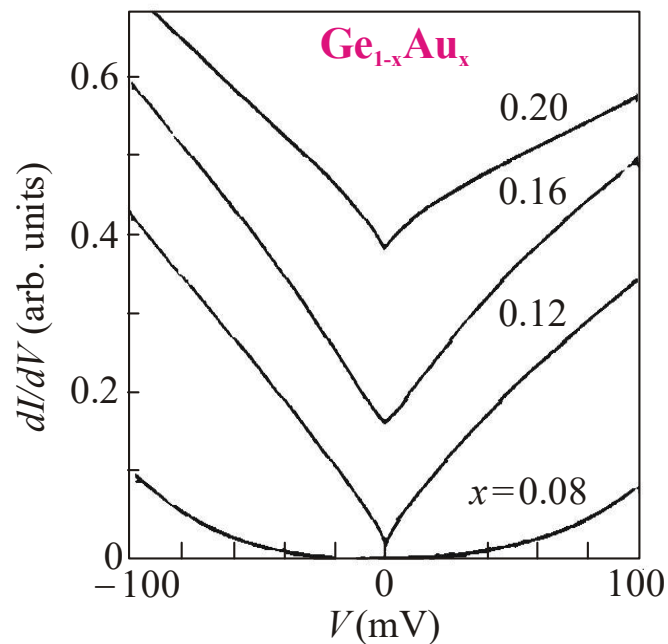
$$g(T, \epsilon) \simeq \begin{cases} g_F, & |\epsilon_F - \epsilon| > \hbar/\tau, \\ g(0, \epsilon), & T < |\epsilon_F - \epsilon| < \hbar/\tau, \\ g(0, \epsilon = T), & |\epsilon_F - \epsilon| < T. \end{cases}$$

Tunneling experiments

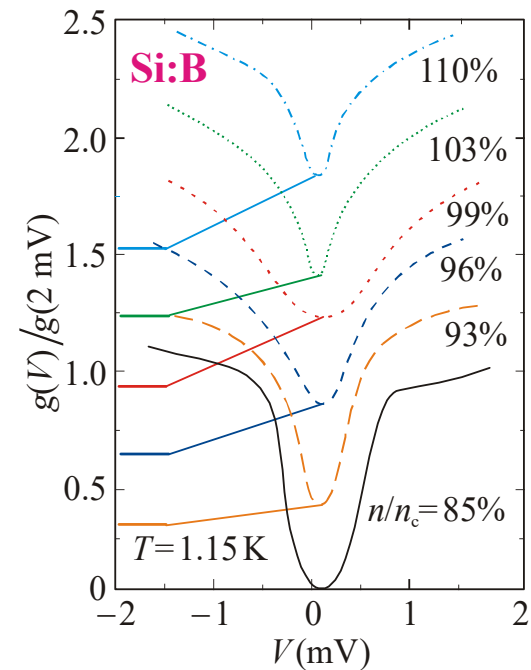


W.L. McMillan, J. Mochel
PRL **46**, 556 (1981)

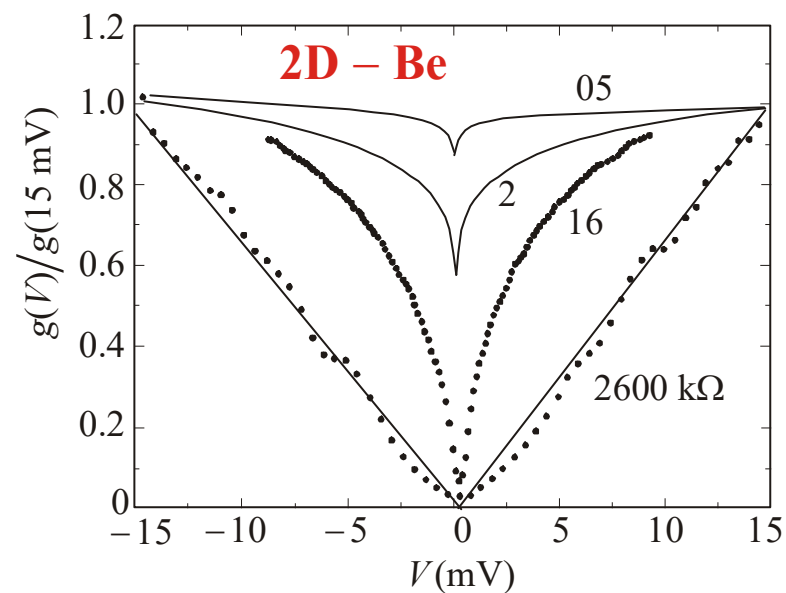
V.Yu. Butko, J.F. DiTisa, P.V. Adams,
PRL **84**, 1543 (2000)



G. Hertel et al.,
PRL **50**, 743 (1983)



J.G. Massey, M. Lee
PRL **77**, 3399 (1996)



Quantum correction to the conductivity due to e-e interaction

*e-e interaction influences transport via correction to the density of states.
The correction is proportional to the probability of e-e collision
within time*

$$\tau_{ee} \sim \hbar/T$$

$$\int_{\tau}^{\hbar/T} \frac{v_F \lambda^2 dt}{(Dt)^{d/2} b^{3-d}}, \quad d = 1, 2, 3$$

=> Altshuler – Aronov effect:

$$L_{ee} \approx \sqrt{D\tau_{ee}} \approx \sqrt{\frac{\hbar D}{T}}$$

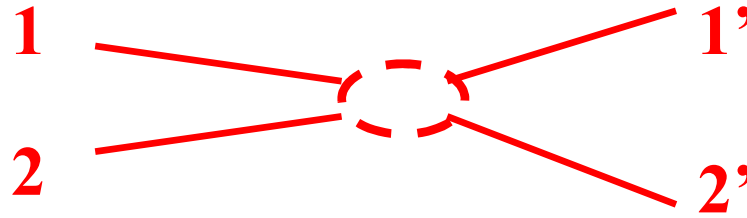
$$\Delta_{ee}\sigma_3 \approx \frac{e^2}{\hbar} \left(\frac{1}{L_{ee}} - \frac{1}{l} \right)$$

$$\Delta_{ee}\sigma_2 \approx -\frac{e^2}{\hbar} \ln L_{ee}/l$$

$$\Delta_{ee}\sigma_1 \approx \frac{e^2}{\hbar} (l - L_{ee})$$

Influence of diffusion on the frequency of ee-collisions

ballistic regime



$$\frac{\hbar}{\tau_e} \sim \frac{T^2}{\epsilon_F}$$

diffusive regime

Size of interaction region $L_{ee} \gg 1/k_F$, momentum transfer is small :

$$q \approx 1/L_{ee} \ll k_F$$

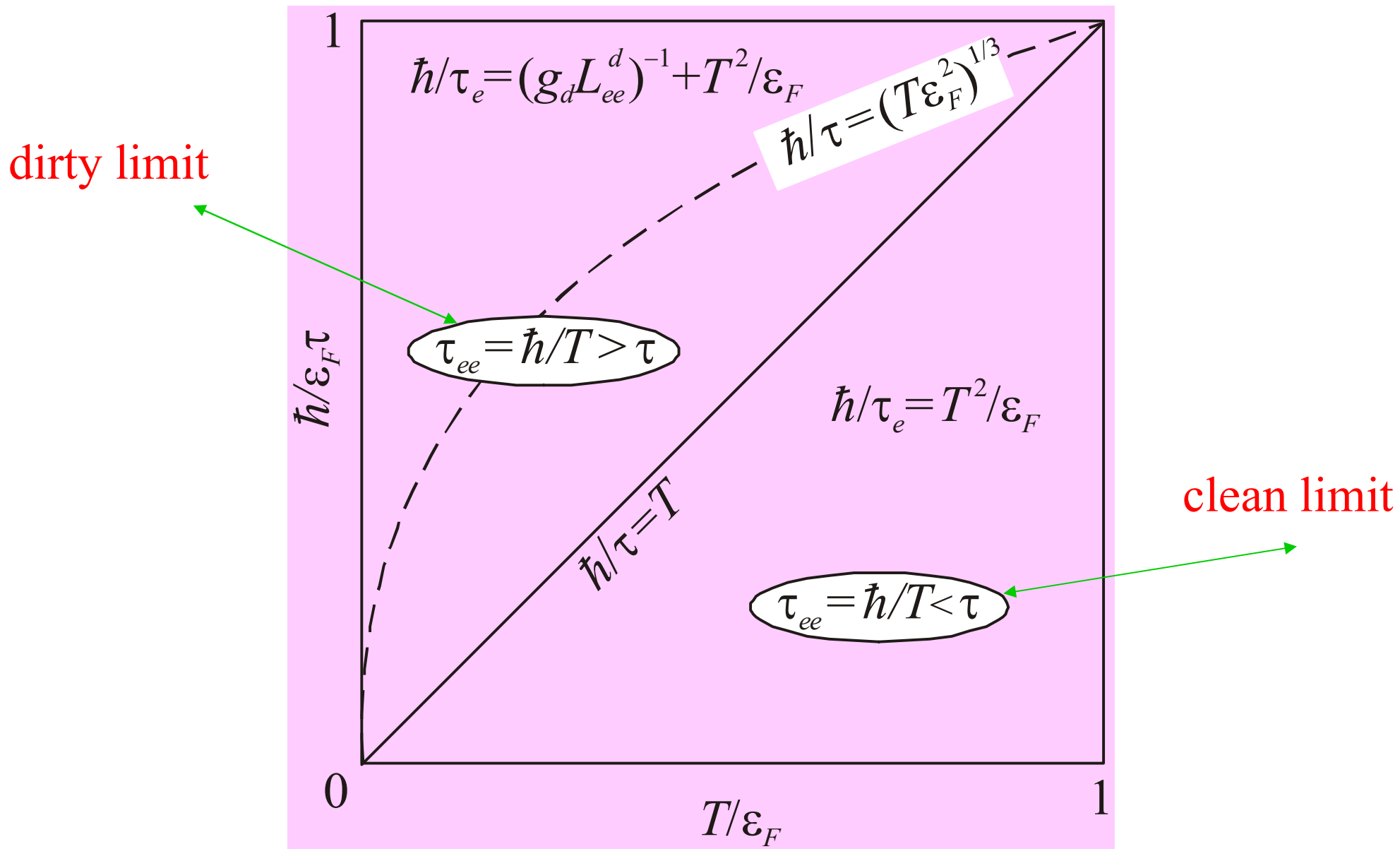
$$\frac{\hbar}{\tau_e} \sim \frac{q^d}{g_d} \sim \frac{1}{g_d L_{ee}^d}$$

, density of states

$$g_d \sim \epsilon_F^{d/2-1} m^{1/2}$$

$$\frac{\hbar}{\tau_e} \sim T^{d/2} \epsilon_F^{1-d} \tau^{-d/2} = \begin{cases} T^{1/2} \tau^{-1/2}, & d = 1 \\ T \epsilon_F^{-1} \tau^{-1}, & d = 2 \\ T^{3/2} \epsilon_F^{-2} \tau^{-3/2}, & d = 3 \end{cases}$$

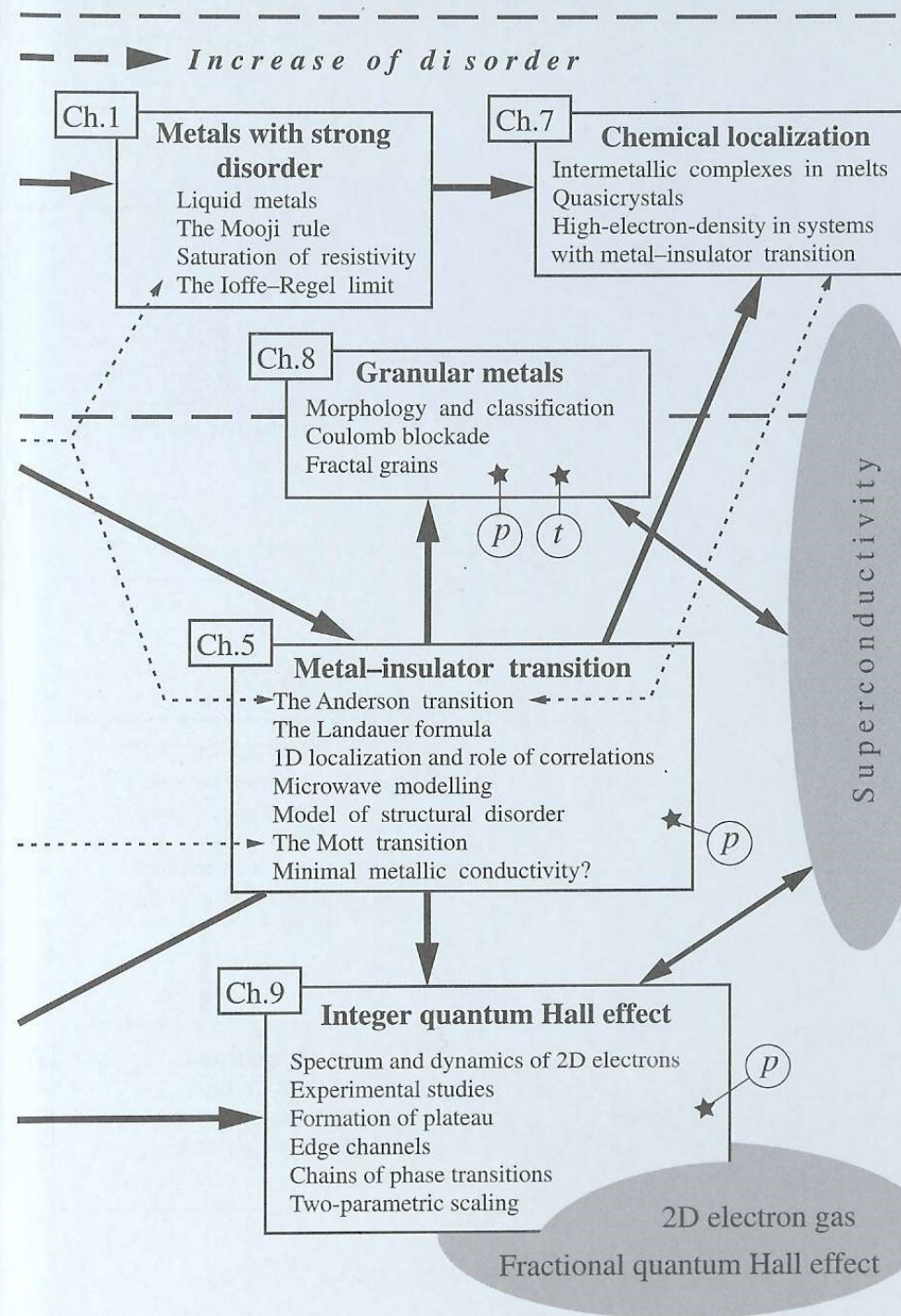
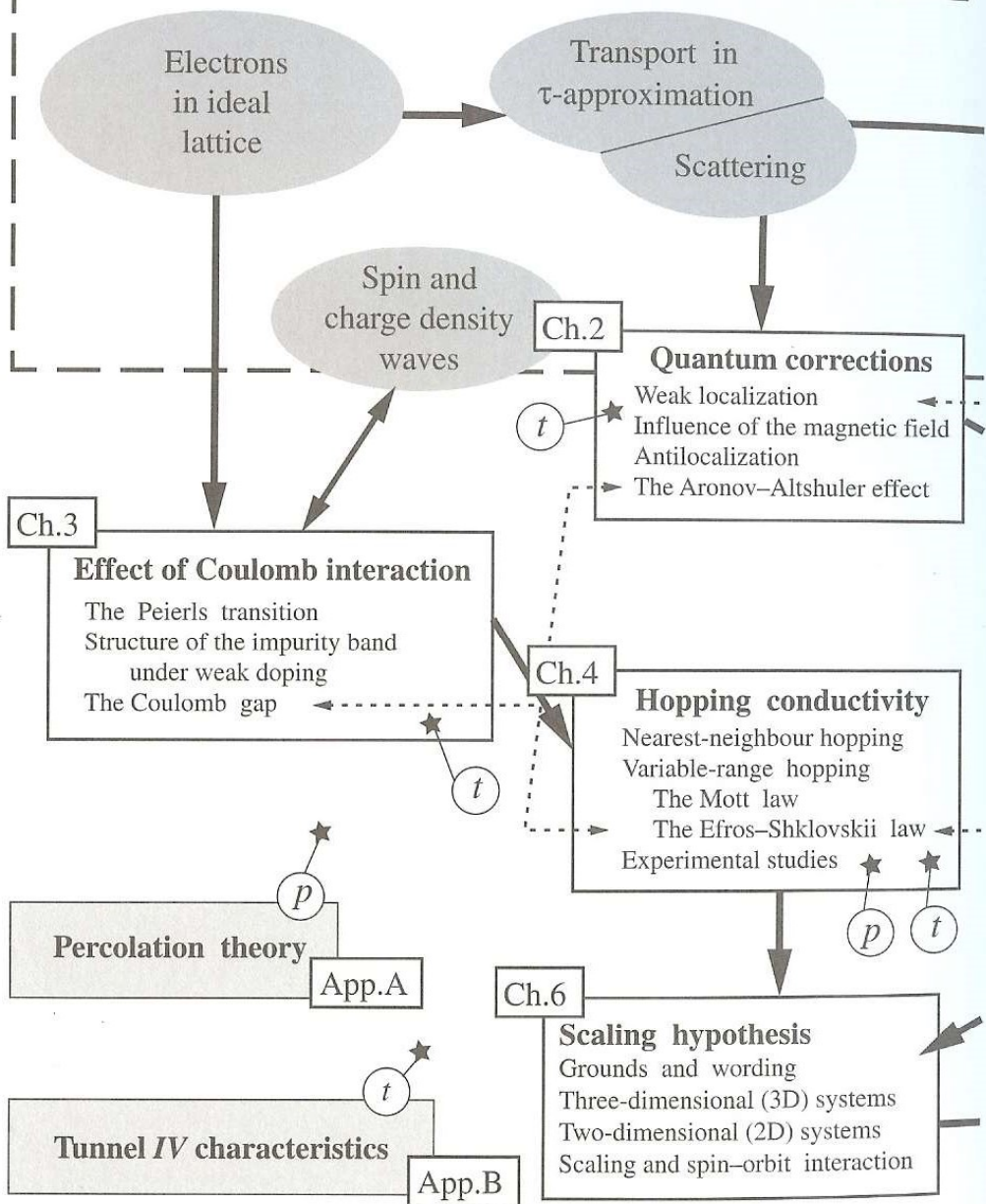
This time constant τ_e controls the weak localization processes









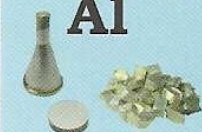























$$\frac{1}{\tau_{eff}} = \frac{1}{\tau} + \frac{1}{\tau_{ph}}$$

Effect of Coulomb Interaction

High-density electron gas



Electrons and disorder in solids
(Schematic guide)

13		14		15		16		17		18	
5	10.811	6	12.011	7	14.007	8	15.999	9	18.998	10	20.180
B		C		N		O		F		Ne	
Boron		Carbon		Nitrogen		Oxygen		Fluorine		Neon	
2.350 79.5	2100 2550(subl.)	3.514 77.2	3547.1 3652	1.251 71	-209.86 -195.82	1.429 60.5	-218.80 -182.96	1.696 70.9	-219.62 -188.14	0.8999 160	-248.61 -246.06
13	26.982	14	28.086	15	30.974	16	32.066	17	35.453	18	39.948
Al		Si		P		S		Cl		Ar	
Aluminium		Silicon		Phosphorus		Sulfur		Chlorine		Argon	
2.70 143.2	660.37 2467	2.33 117.6	1412 2355	1.823 110.5	44.10 280.5	1.958 103.5	119.6 444.67	3.214 99.4	-100.98 -34.06	1.784 174	-189.37 -185.86
31	69.723	32	72.61	33	74.922	34	78.96	35	79.904	36	83.80
Ga		Ge		As		Se		Br		Kr	
Gallium		Germanium		Arsenic		Selenium		Bromine		Krypton	
5.904 122.1	29.78 2403	5.323 122.5	937.4 2830	5.727 124.5	817 615(subl.)	4.79 140	217 685	3.123 114.5	-7.25 58.78	3.749 189	-157.20 -153.35
49	114.82	50	118.71	51	121.76	52	127.60	53	126.90	54	131.29
In		Sn		Sb		Te		I		Xe	
Indium		Tin		Antimony		Tellurium		Iodine		Xenon	
7.31 162.6	156.61 2080	7.29 140.5	231.97 2270	6.684 145	630.5 1635	6.25 143.2	449.5 990	4.940 133.3	113.60 184.35	5.897 218	-111.80 -108.10
81	204.38	82	207.2	83	208.98	84	209.98	85	209.99	86	222.02
Tl		Pb		Bi		Po		At		Rn	
Thallium		Lead		Bismuth		Polonium		Astatine		Radon	
11.85 170	303.10 1457	11.342 175	327.5 1740	9.79 154.5	271.3 1560	9.196 164	254 962	8.75 145	302 370	9.73 -	-71.1 -61.8

2 4.0026

He

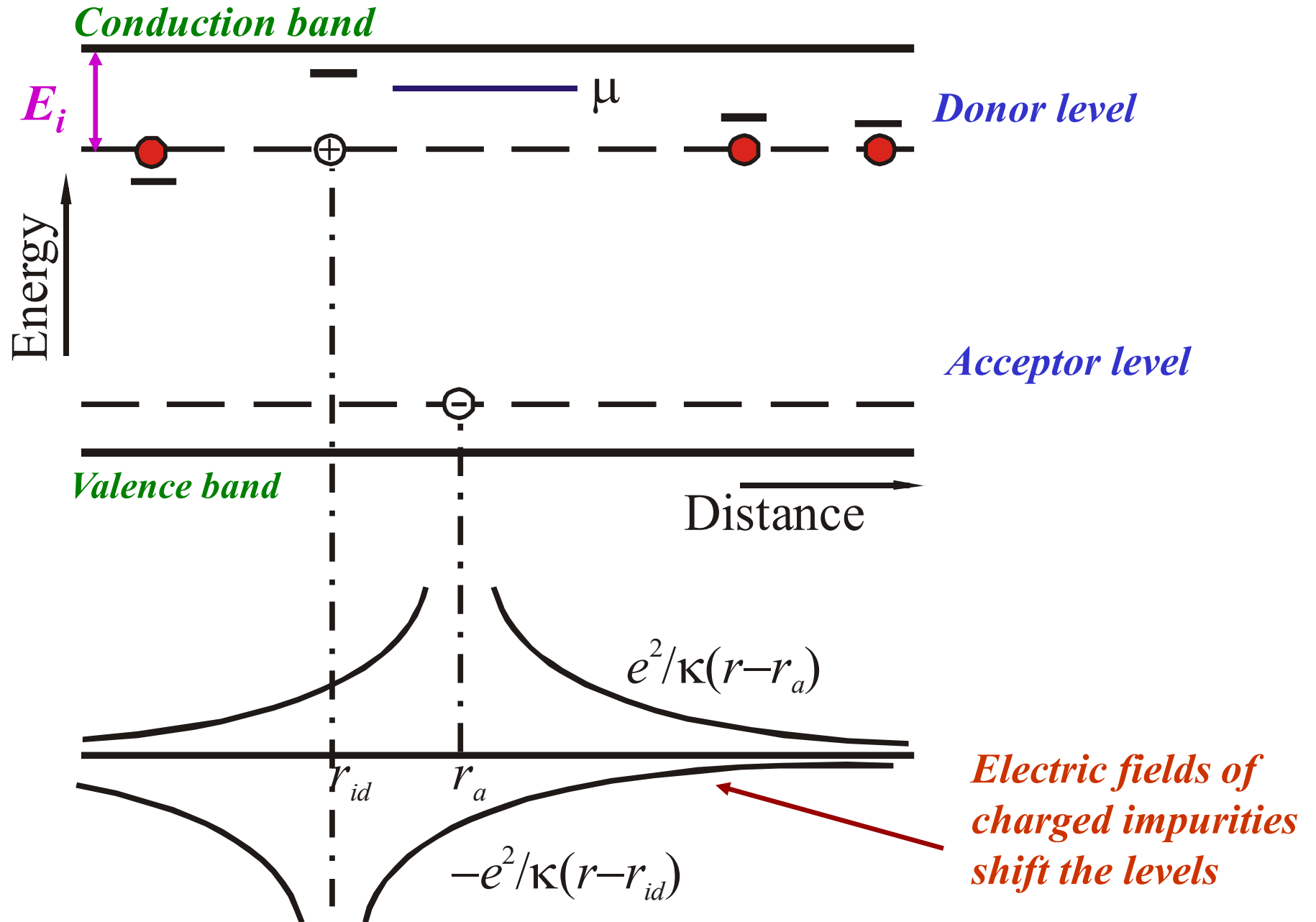
Helium

0.1785
128



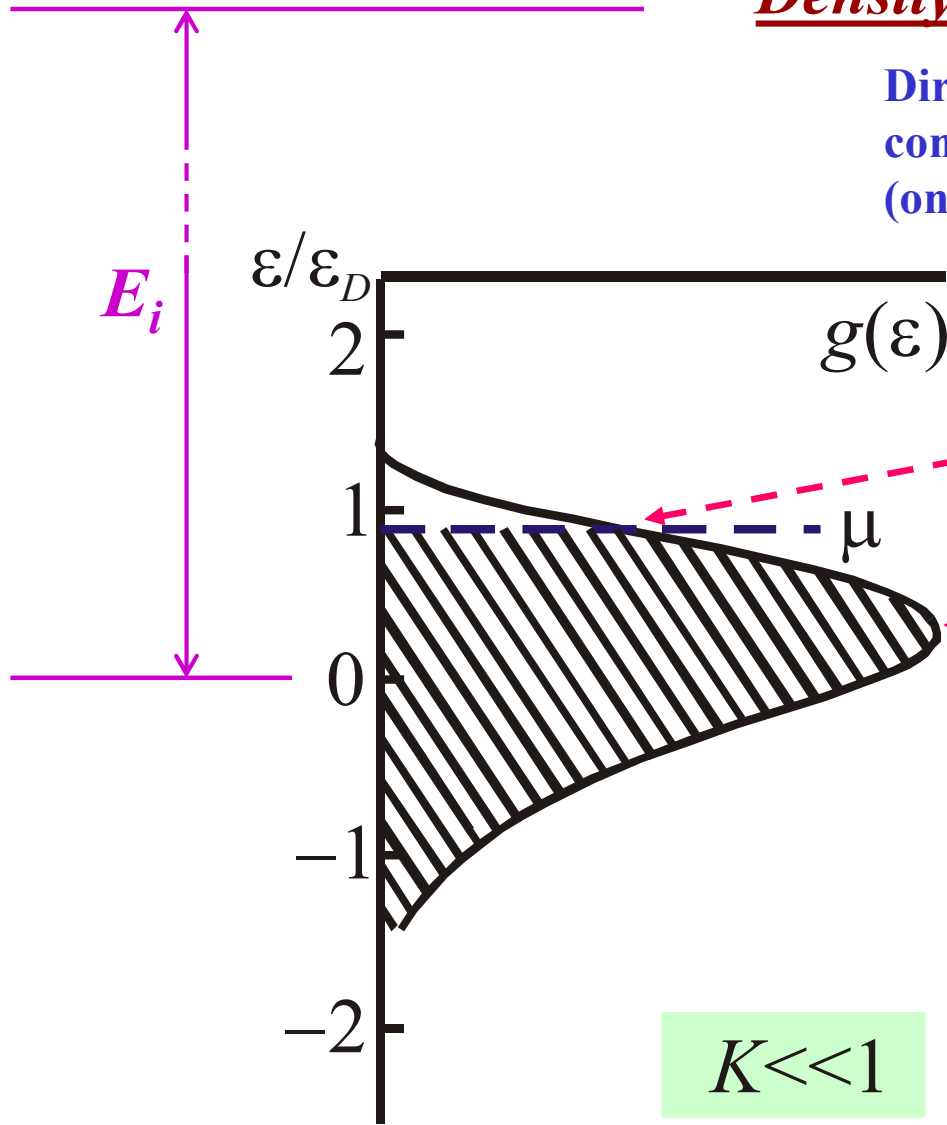
-268.94

Impurity band



Density of states in the impurity band

Direct overlap of wave functions is negligible
 conduction band is not formed \Rightarrow insulator at $T=0$
 (only thermally activated hopping is possible)



Bohr radius

$$a_B = \frac{\kappa \hbar^2}{m^* e^2}$$

ionization energy $E_i = \frac{e^2}{2a_B \kappa}$

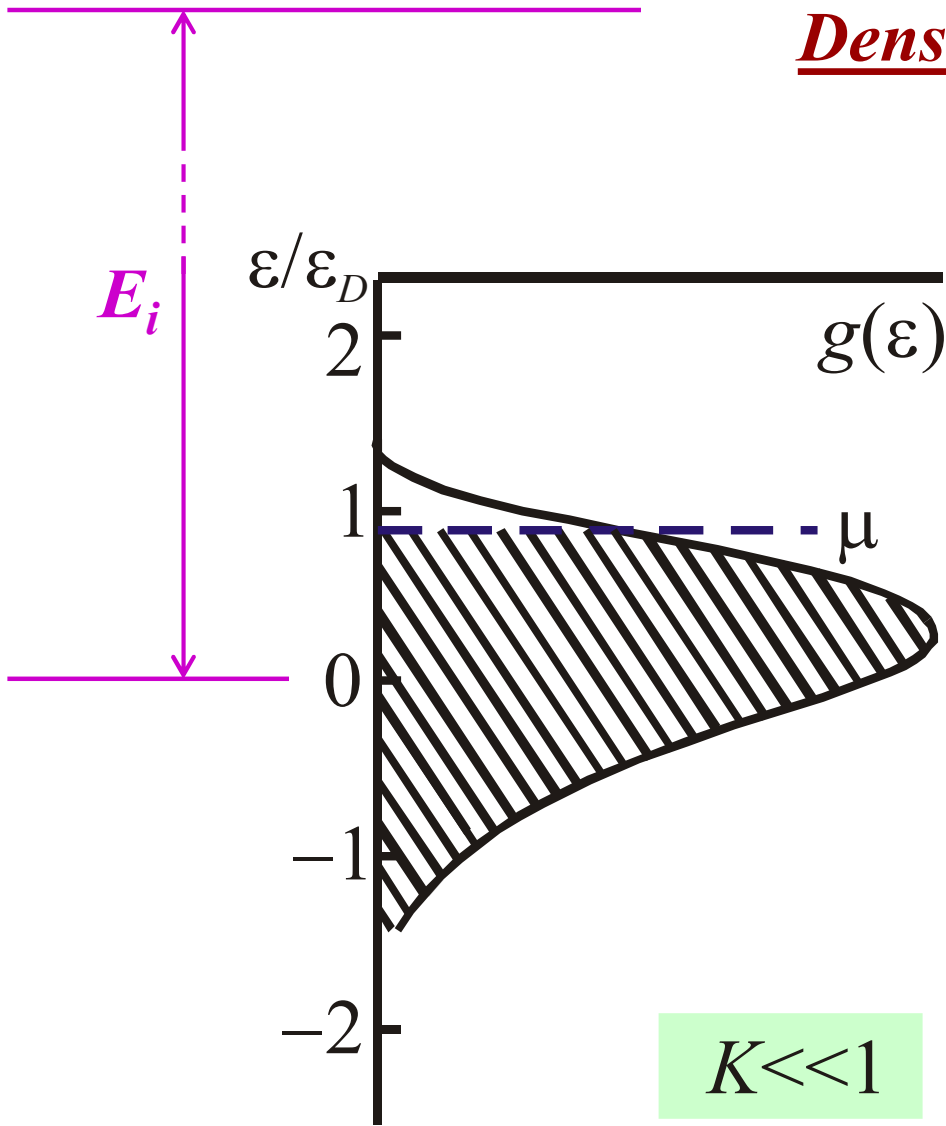
low doping:

$$N a_B^3 \ll 1, \Rightarrow \varepsilon_D \ll E_i$$

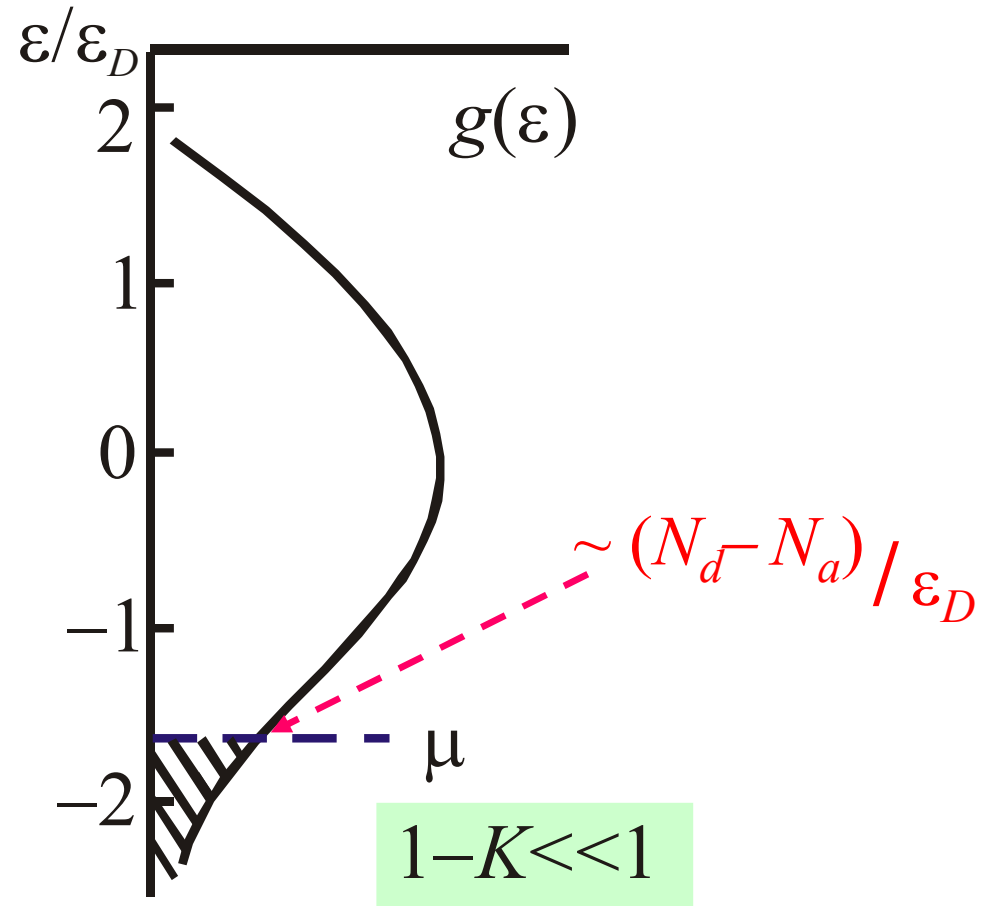
compensation $K = N_a / N_d$

band width $\varepsilon_D = \frac{e^2}{\kappa} N_d^{1/3}$

Density of states in the impurity band

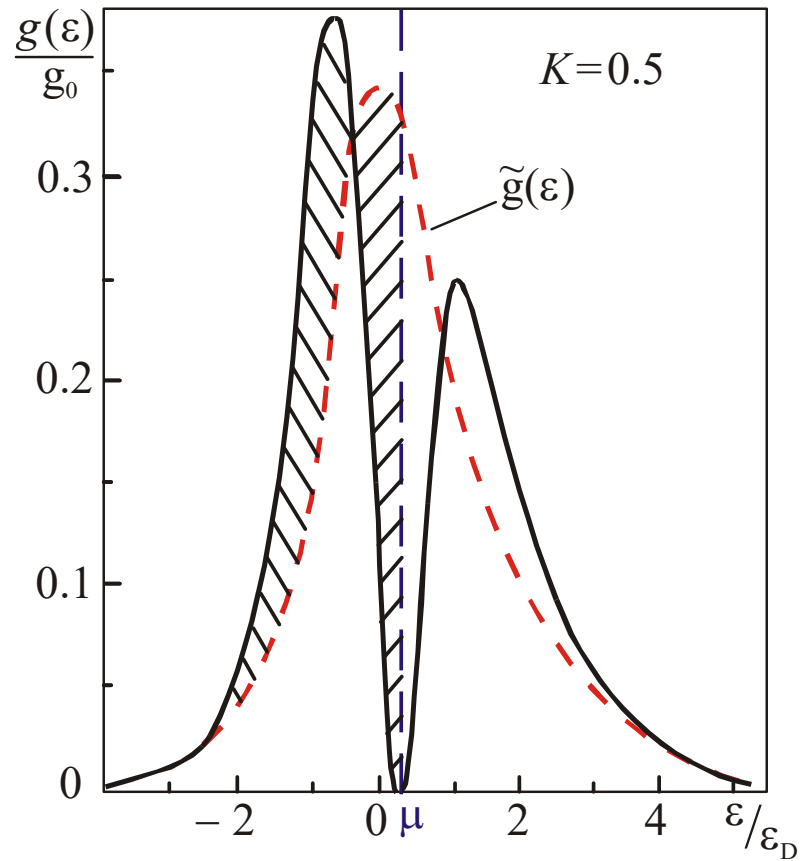


Weak compensation



Strong compensation

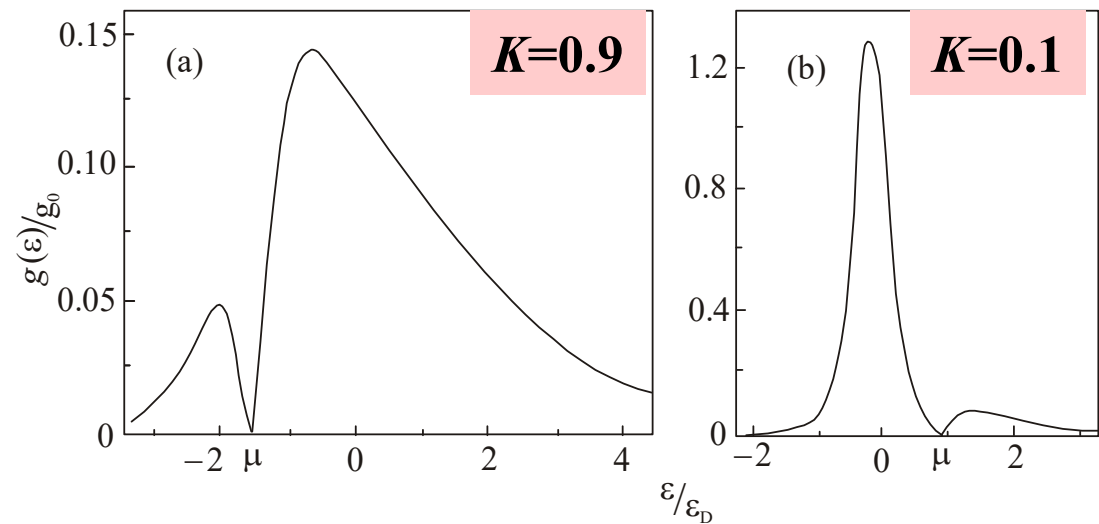
Coulomb gap

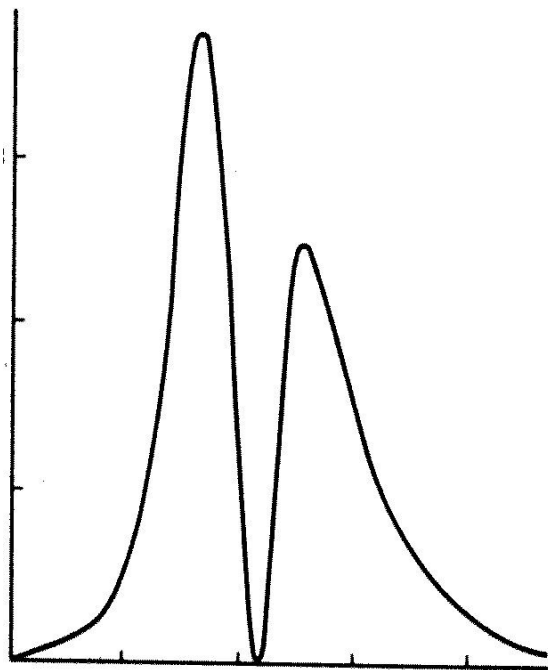
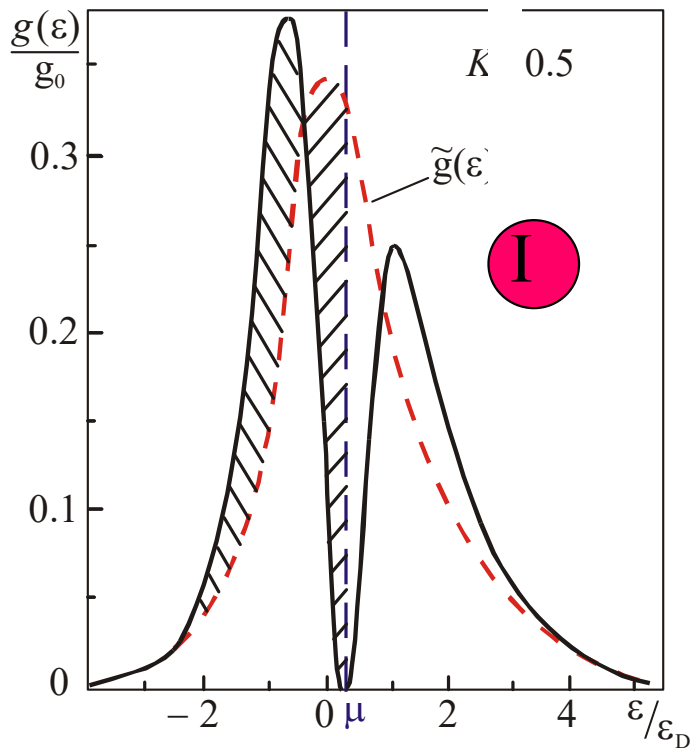


Energy of occupied states decreased

Energy of empty states increased

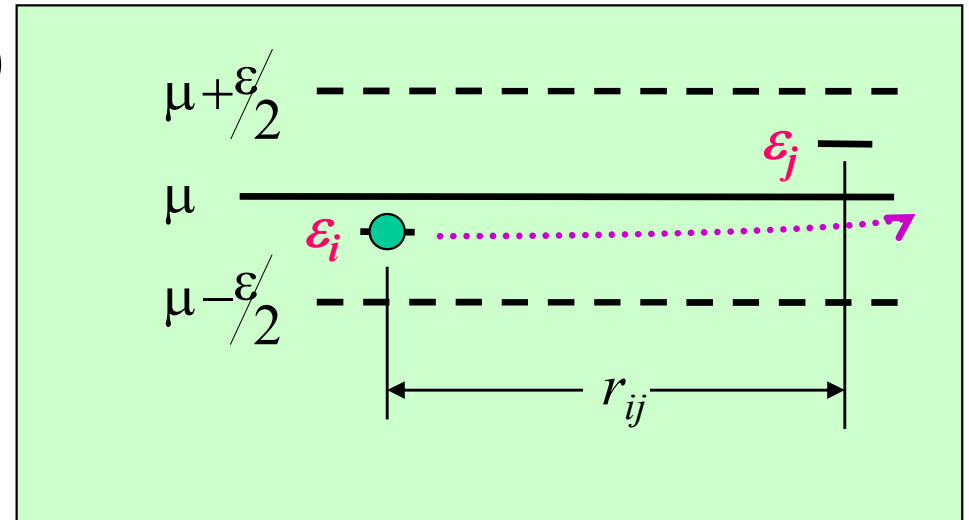
A.L. Efros, N.V. Lien, B.I. Shklovskii
J. Phys. C **12**, 1023 (1979)





Coulomb gap

II



$$\varepsilon_j - \varepsilon_i - \frac{e^2}{\kappa r_{ij}} \geq 0 \quad \rightarrow \quad \frac{e^2}{\kappa r_{ij}} \leq \varepsilon$$

3D

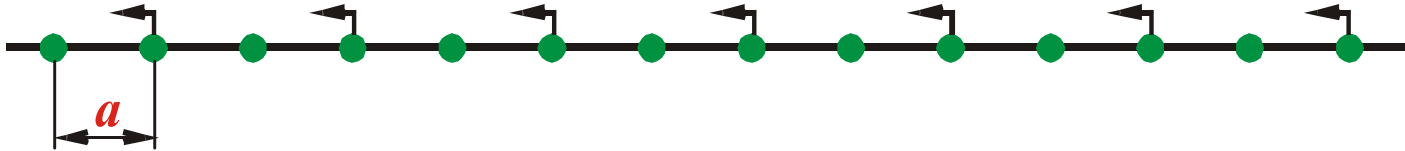
$$N(\varepsilon) = r_{ij}^{-3} \leq \left(\frac{\kappa \varepsilon}{e^2} \right)^3, \quad g(\varepsilon) = \frac{\partial N}{\partial \varepsilon} \propto \frac{\varepsilon^2 \kappa^3}{e^6}$$

2D

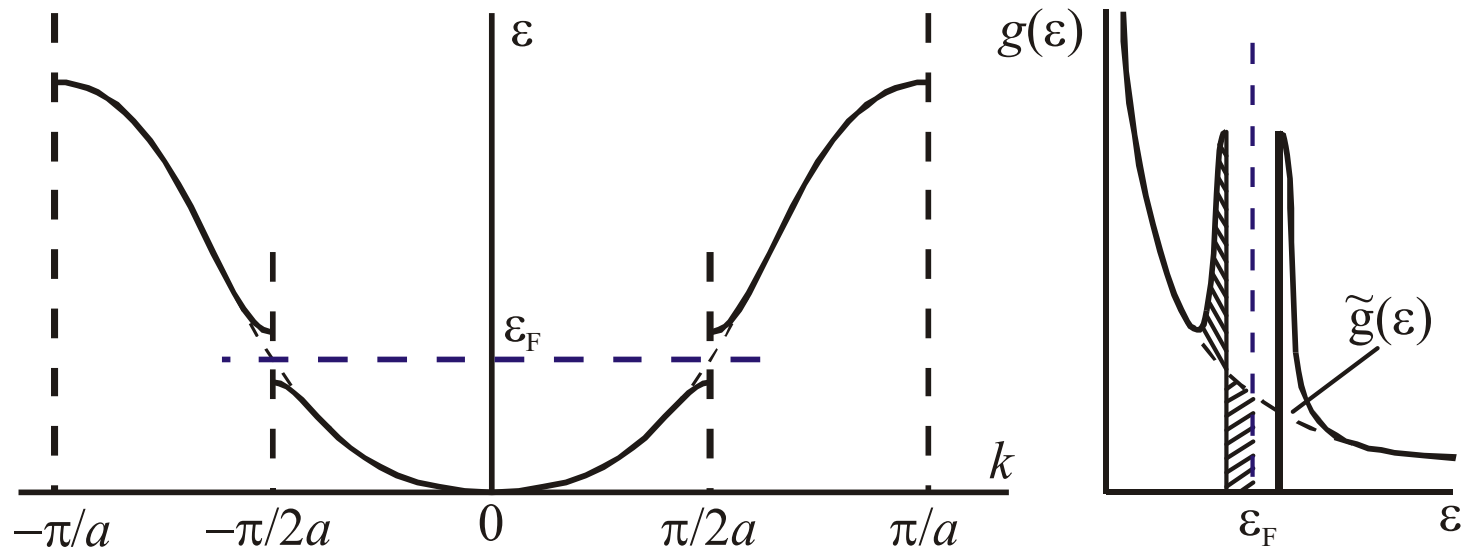
$$N(\varepsilon) = r_{ij}^{-2} \leq \left(\frac{\kappa \varepsilon}{e^2} \right)^2, \quad g(\varepsilon) = \frac{\partial N}{\partial \varepsilon} \propto \frac{|\varepsilon| \kappa^2}{e^4}$$

Another example of electronic phase transition:

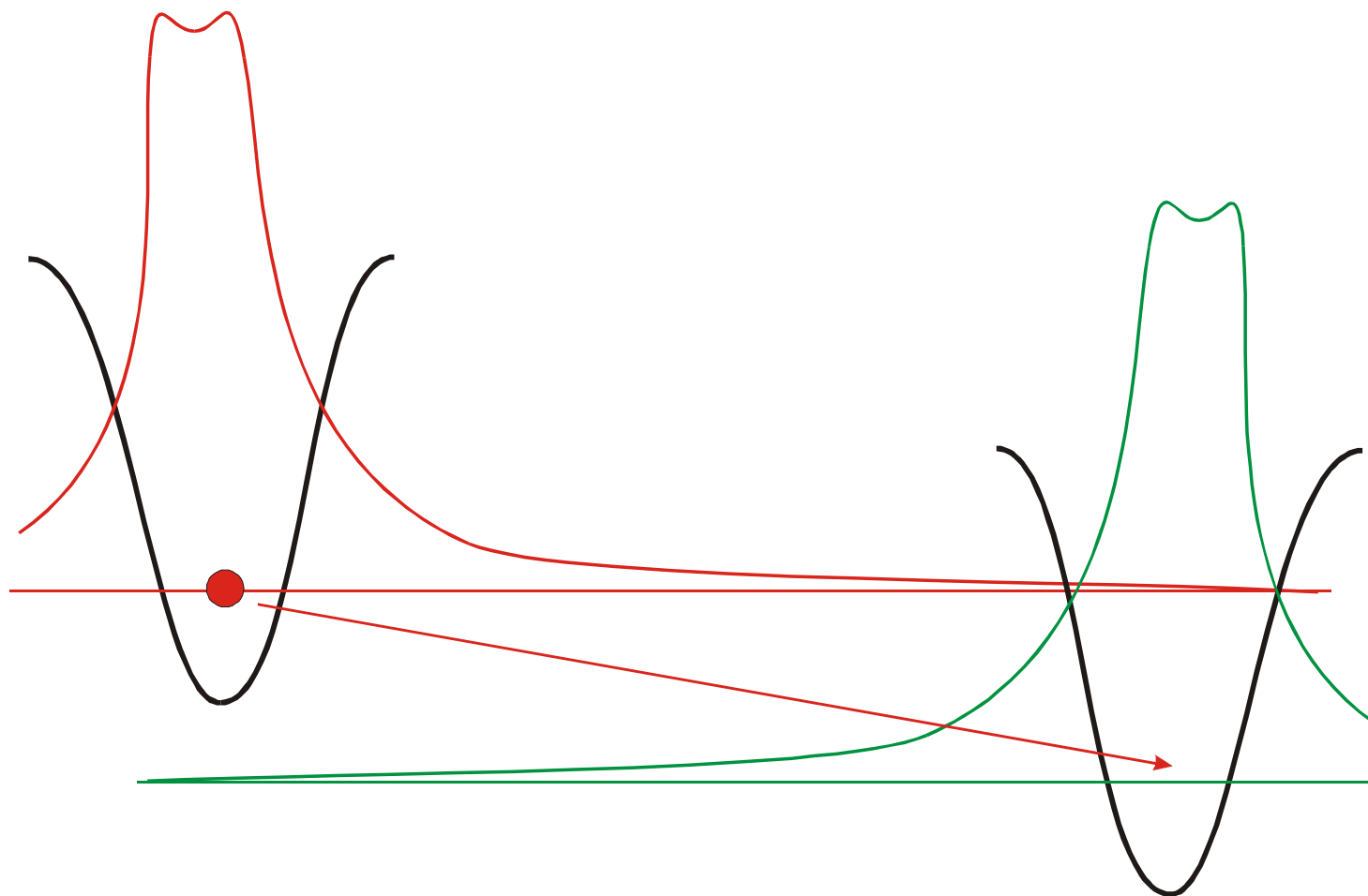
Peierls transition



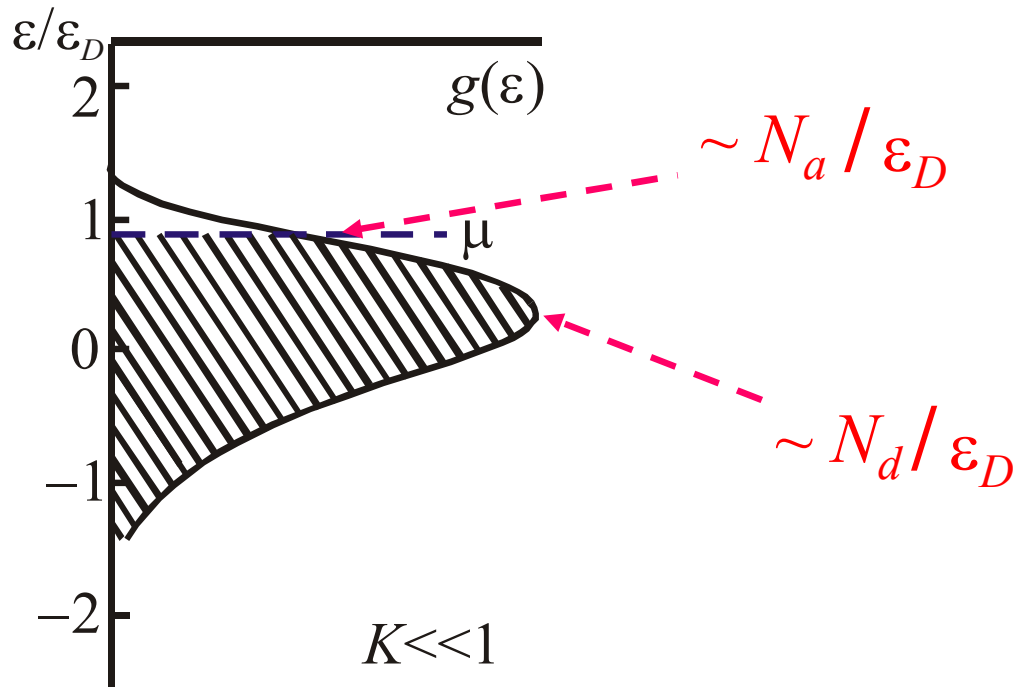
Period doubling $a \rightarrow 2a$



Hopping conductivity



Main characteristics of an impurity band



Bohr radius

$$a_B = \frac{\kappa \hbar^2}{m^* e^2}$$

band width $\varepsilon_D = \frac{e^2}{\kappa} N_d^{1/3}$

low doping regime: $a_B \ll r_{ij}$
 where r_{ij} is the distance
 between neighboring sites

Compensation factor

$$K = N_a / N_d$$

$$Na_B^3 \ll 1, \Rightarrow \varepsilon_D \ll E_i$$

Hydrogen atom: attractive Coulomb potential $U = -(e^2/r)$

the Bohr radius $a_B = \hbar^2 / me^2$

$$\psi_n(r) \rightarrow C(n)r^{n-1} \exp(-r/na_B) \text{ as } r \rightarrow \infty, \quad (n = 1, 2, 3, \dots)$$

In the ground state ($n=1$) localization length equals a_B

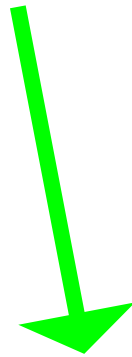
Hopping probability

$$\frac{1}{\tau_{ij}} \propto F(\varphi_{ij}, f_i, f_j) \left| \int \psi_j^* e^{iqr} \psi_i d^3 r \right|^2$$

$$f_i = \left(\exp \frac{\varepsilon_i - \mu}{T} + 1 \right)^{-1}$$

$$\varphi_{ij} = \left(\exp \frac{\varepsilon_{ij}}{T} - 1 \right)^{-1}$$

$$\varepsilon_{ij} = \varepsilon_j - \varepsilon_i$$

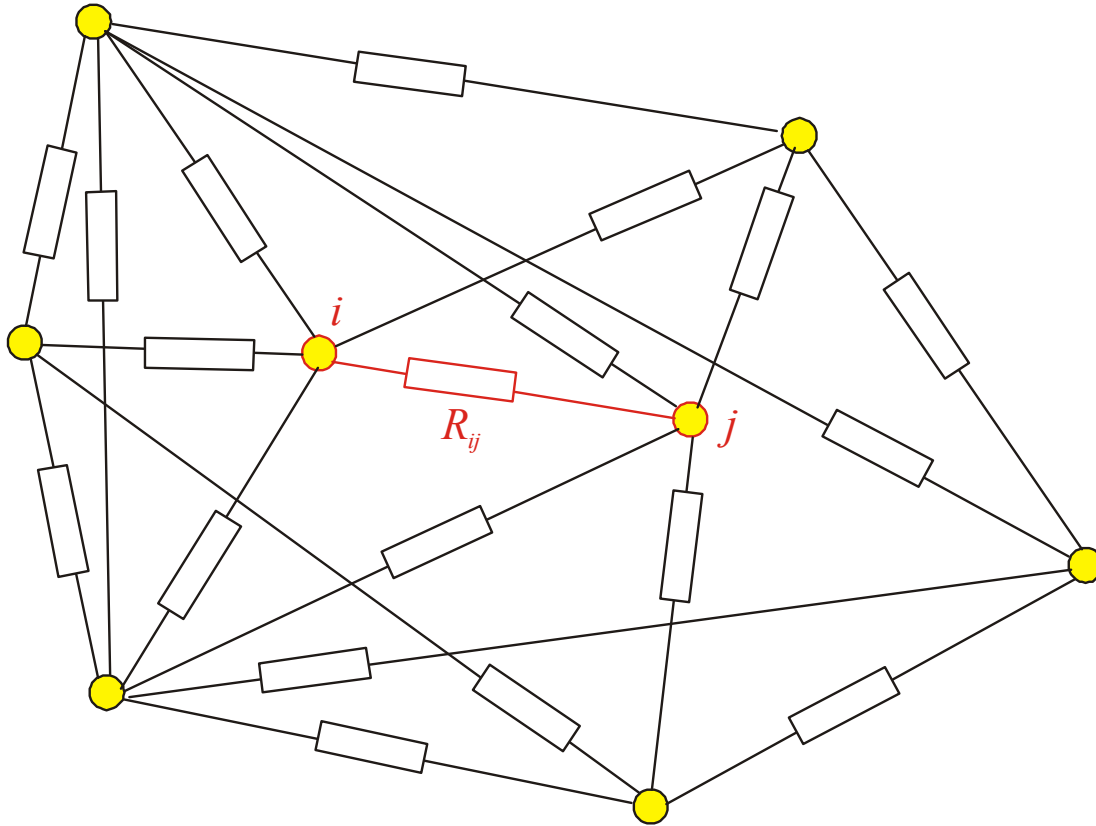


$$e^{-\varepsilon_{ij} / T}$$



$$e^{-2r_{ij} / a_B}$$

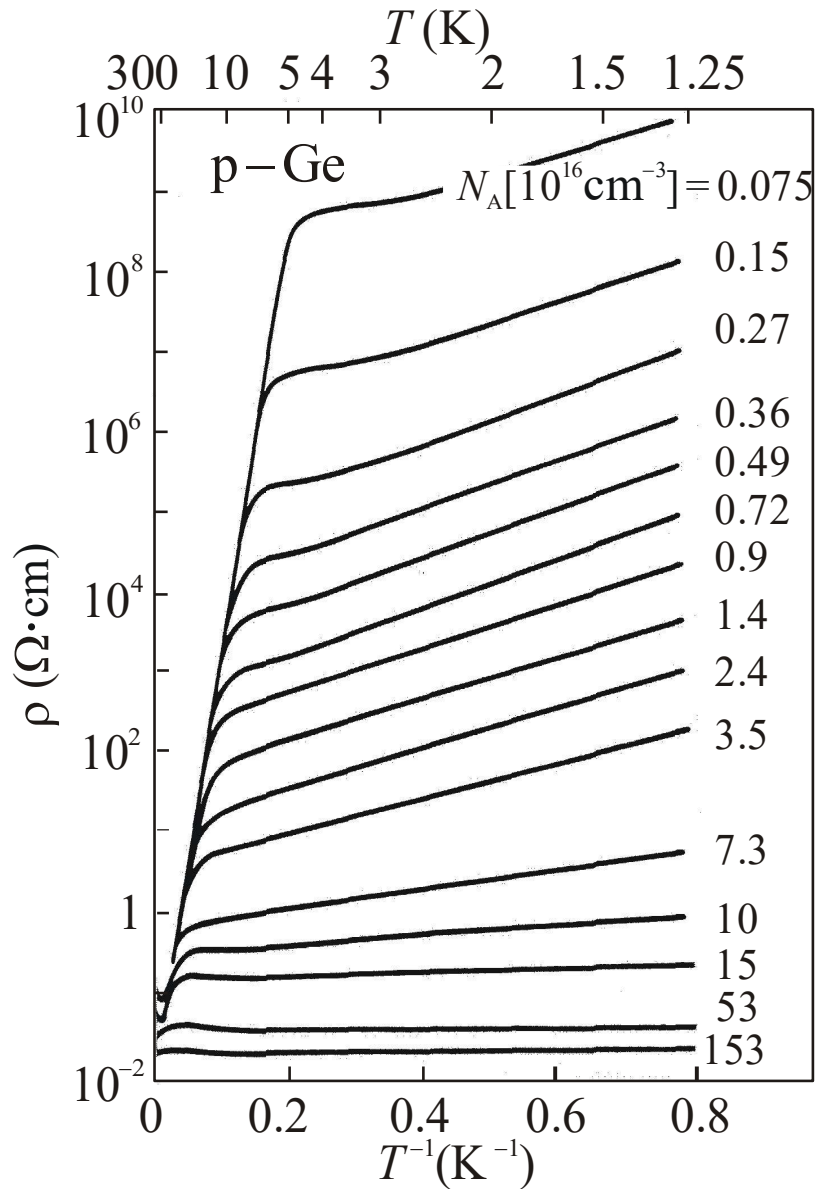
Abrahams-Miller net



$$R_{ij} = R_0 e^{u_{ij}}$$

$$u_{ij} = \frac{2r_{ij}}{a_B} + \frac{\varepsilon_{ij}}{T}$$

Experiment



H.Fritzsche, M.Guevas, PR **119**, 1238 (1960)

Neutron-irradiated Ge

As a result of nuclear reaction

one of Ge isotopes \rightarrow Ga

another Ge isotope \rightarrow As

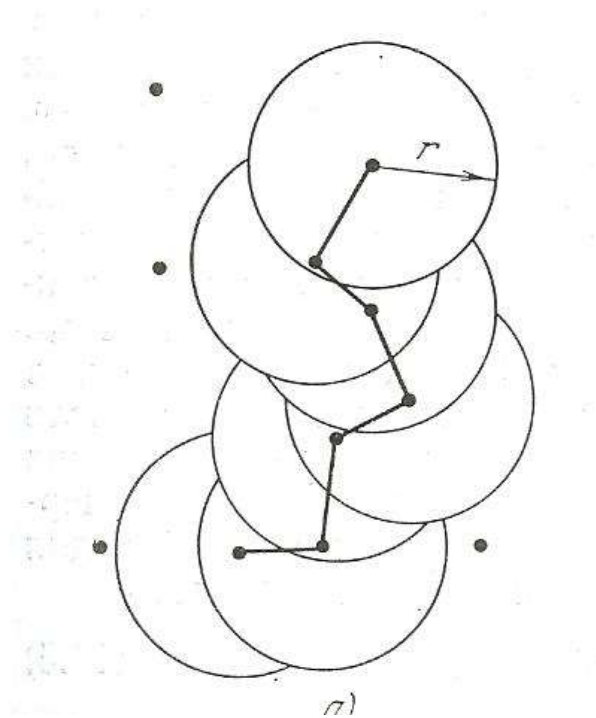
Ga –acceptors, As – donors

$K = 0.4$ is fixed

while N_a is a function of irradiation time

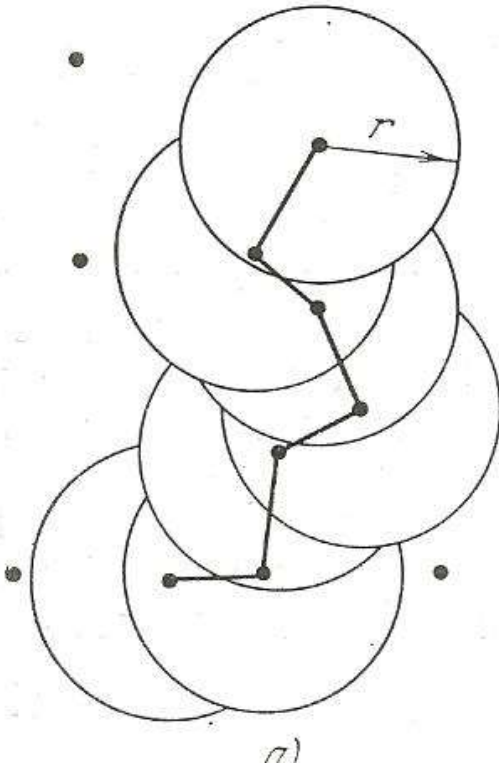
Basic ideas of percolation theory

- *The nodes are introduced which are characterized by concentration n and the radius r of interaction between neighboring nodes*



- *Percolation is the problem of global connectivity across the whole sample via connected nodes*
- *In the case of electrons the interaction radius r is controlled by α_B*

Percolation theory: Random nodes



The percolation threshold depends on a number of nodes within the interaction radius r

The number of nodes in a sphere with radius r

$$\frac{4\pi}{3} r^3 n$$

where n is the concentration of nodes

Numerical results for critical concentration n_c

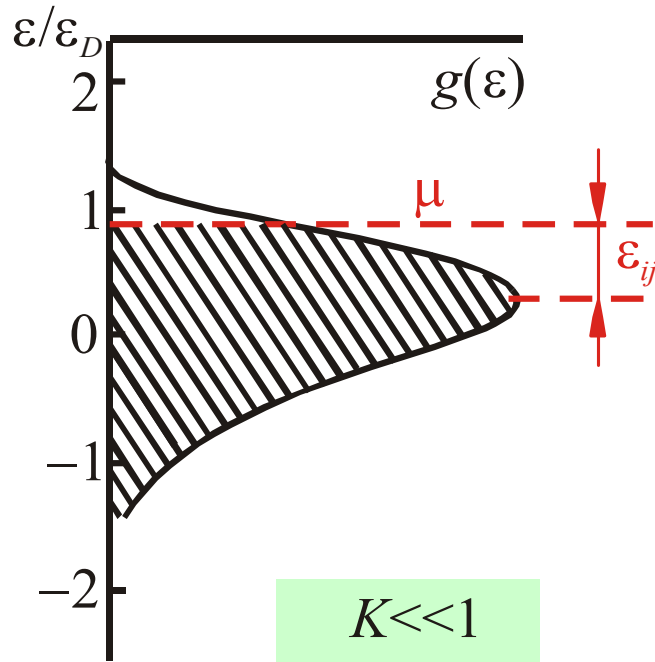
3D

$$\frac{4\pi}{3} r_c^3 n_c = B_c^{(3)} = 2.7$$

2D

$$\pi r_c^2 n_c = B_c^{(2)} = 4.4$$

Nearest neighbour hopping



$K \ll 1$

$$e^{-\frac{\epsilon_{ij}}{T}}$$

Abrahams-Miller net

$$R_{ij} = R_0 e^{u_{ij}}$$

$$u_{ij} = \frac{2r_{ij}}{a_B} + \frac{\epsilon_{ij}}{T}$$

T-dependent factor is the same for all transitions

$$\frac{4\pi}{3} r_c^3 n = B_c = 2.7 \quad \Rightarrow \quad r_c = 0.865 n^{-1/3}$$

Dependence on concentration: r_{ij} in the Abrahams-Miller relation is replaced by r_c

\Rightarrow

$$\rho = \rho_0 \exp\left(\frac{1.73}{n^{1/3} a_B}\right)$$

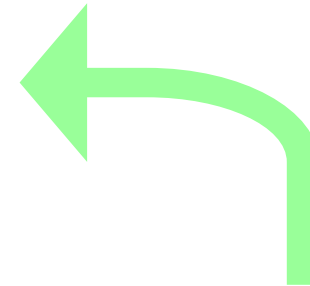
Experiment

Percolation threshold

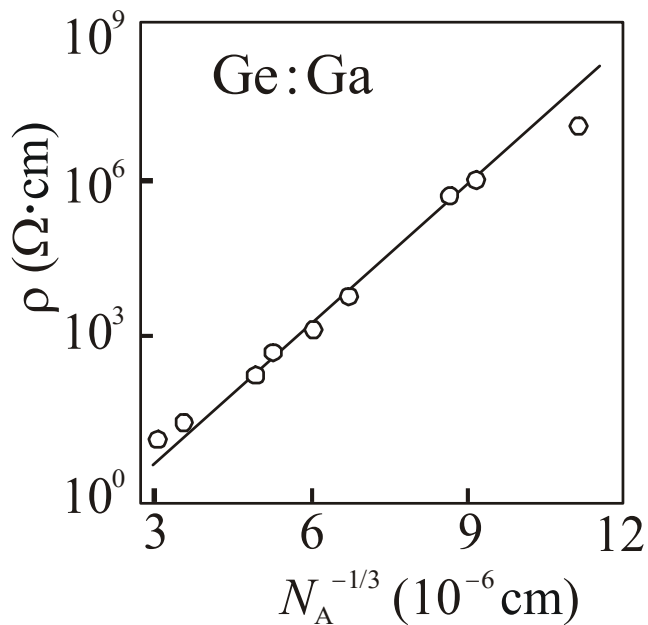
$$\frac{4\pi}{3} r_c^3 n = B_c = 2.7 \quad \Rightarrow$$

$$r_c = 0.865 n^{-1/3}$$

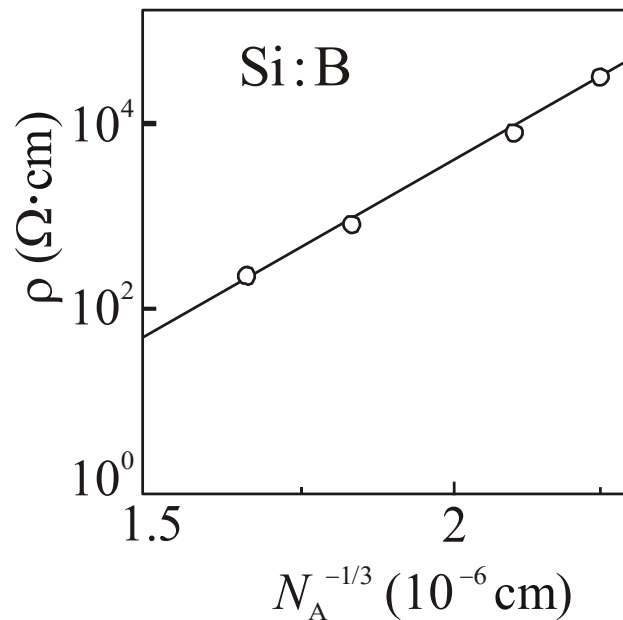
$$\rho = \rho_0 \exp\left(\frac{1.73}{n^{1/3} a_B}\right)$$



H.Fritzsche, M.Guevas,
Phys. Rev. **119**, 1238 (1960)
Ga -acceptors, As - donors

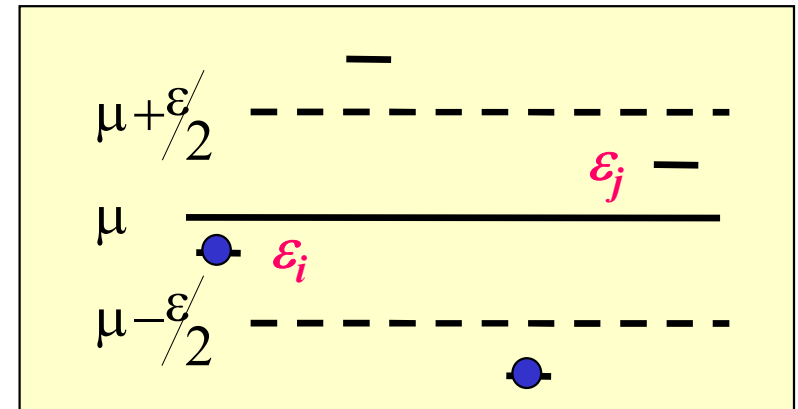
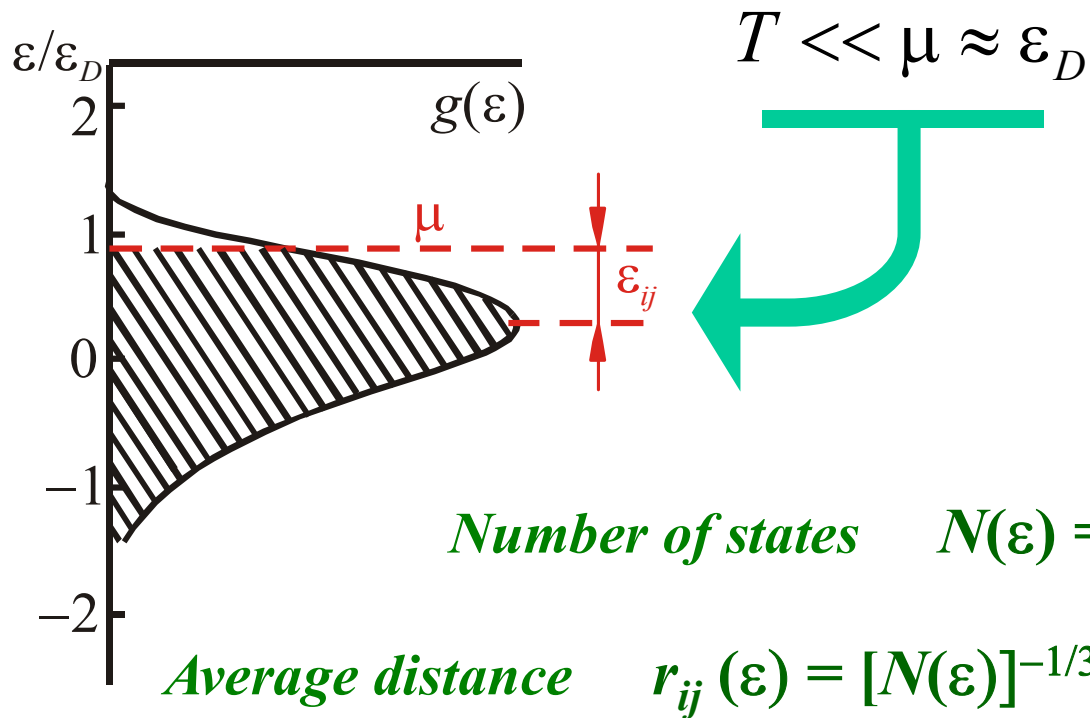


R. Ray, H.Fan,
Phys. Rev. **121**, 768 (1961)



n-GaAs	1.7	1.88	1.9
n-InP	1.9		
p-Ge	1.9	1.75	2.0
p-Si	1.8		

Variable range hopping: Mott law



parameter u_{ij} of Miller–Abrahams net

$$u_{ij} = \frac{2}{a_B [N(\varepsilon)]^{1/3}} + \frac{\varepsilon}{T} = \frac{2}{g_\mu^{1/3} a_B \varepsilon^{1/3}} + \frac{\varepsilon}{T}$$

u_{ij} depends on ε and reaches its minimum when $\frac{d}{d\varepsilon} u_{ij}(\varepsilon) = 0$

$$\Rightarrow \varepsilon_{\min} = \left(\frac{T}{a_B g_\mu^{1/3}} \right)^{3/4} = (T^3 T_{Mott})^{1/4}, \quad T_{Mott} = (a_B^3 g_\mu)^{-1}$$

Mott law

Average hopping length $\bar{r} = \bar{r}_{ij}(\epsilon_{\min})$

$$\bar{r} = (g_{\mu} \epsilon_{\min})^{-1/3} = a_B \left(\frac{T_{Mott}}{T} \right)^{1/4}$$

Resistance

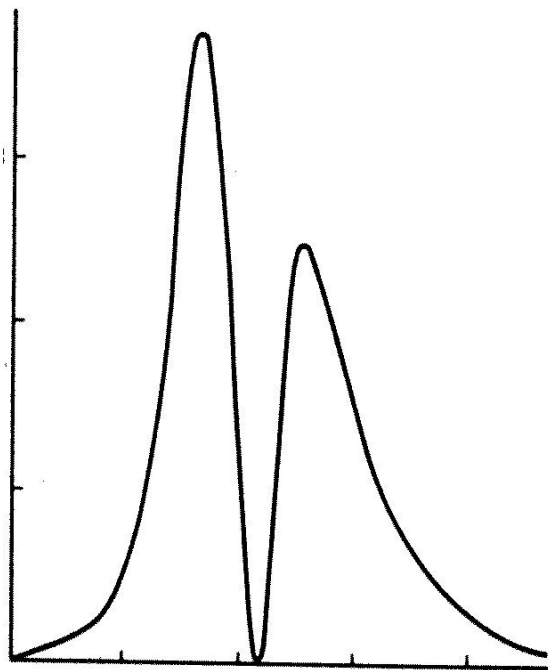
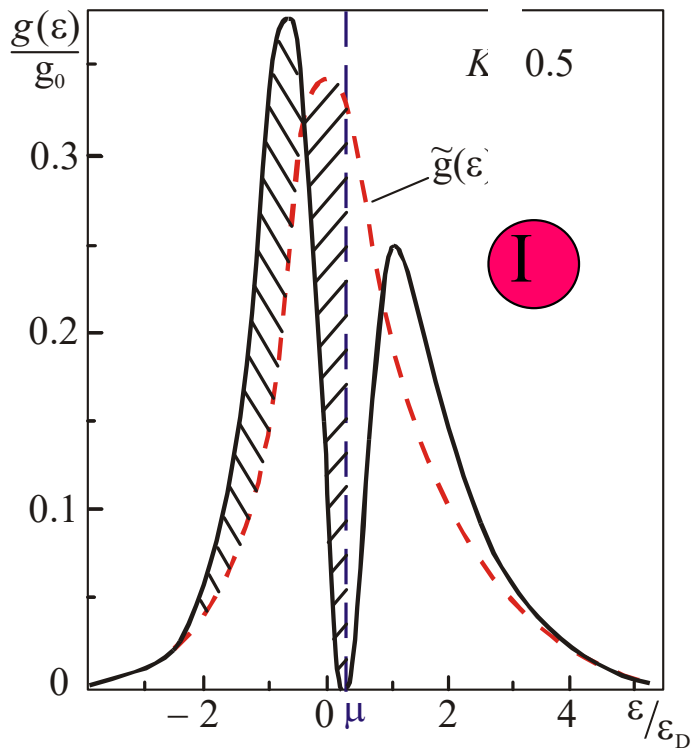
$$\rho = \rho_0 \exp \left(\frac{T_{Mott}}{T} \right)^{1/4} \quad (d = 3)$$

For d=2

$$r_{ij} = [N(\epsilon)]^{-1/2}, \quad u_{ij} = \frac{2}{g_{\mu}^{1/2} a_B \epsilon^{1/2}} + \frac{\epsilon}{T},$$

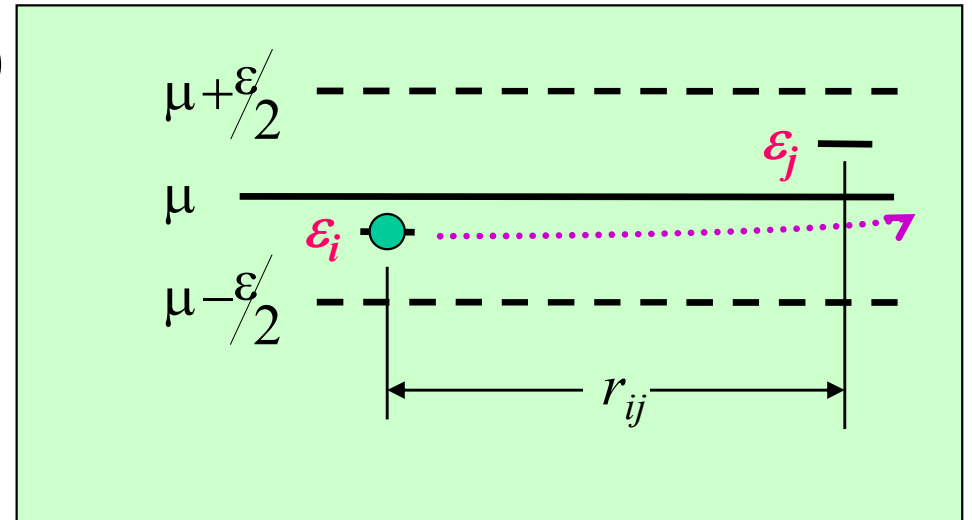
$$\epsilon_{\min} = \left(\frac{T}{g_{\mu}^{1/2} a_B} \right)^{2/3} = (T^2 T_{Mott})^{1/3}, \quad T_{Mott} = (g_{\mu} a_B^2)^{-1}$$

$$\rho = \rho_0 \exp \left(\frac{T_{Mott}}{T} \right)^{1/3} \quad (d = 2)$$



Coulomb gap

II



$$\varepsilon_j - \varepsilon_i - \frac{e^2}{\kappa r_{ij}} \geq 0 \quad \rightarrow \quad \frac{e^2}{\kappa r_{ij}} \leq \varepsilon$$

3D

$$N(\varepsilon) = r_{ij}^{-3} \leq \left(\frac{\kappa \varepsilon}{e^2} \right)^3, \quad g(\varepsilon) = \frac{\partial N}{\partial \varepsilon} \propto \frac{\varepsilon^2 \kappa^3}{e^6}$$

2D

$$N(\varepsilon) = r_{ij}^{-2} \leq \left(\frac{\kappa \varepsilon}{e^2} \right)^2, \quad g(\varepsilon) = \frac{\partial N}{\partial \varepsilon} \propto \frac{|\varepsilon| \kappa^2}{e^4}$$

Variable range hopping: Shklovskii-Efros

Coulomb gap

$$g(\varepsilon) = \left(\frac{\kappa}{e^2} \right)^d |\varepsilon|^{d-1}, \quad g(0) = 0$$

Number of states in ε - interval near Fermi level

$$N(\varepsilon) = \left(\frac{\kappa \varepsilon}{e^2} \right)^d$$

Same procedure as in the Mott case

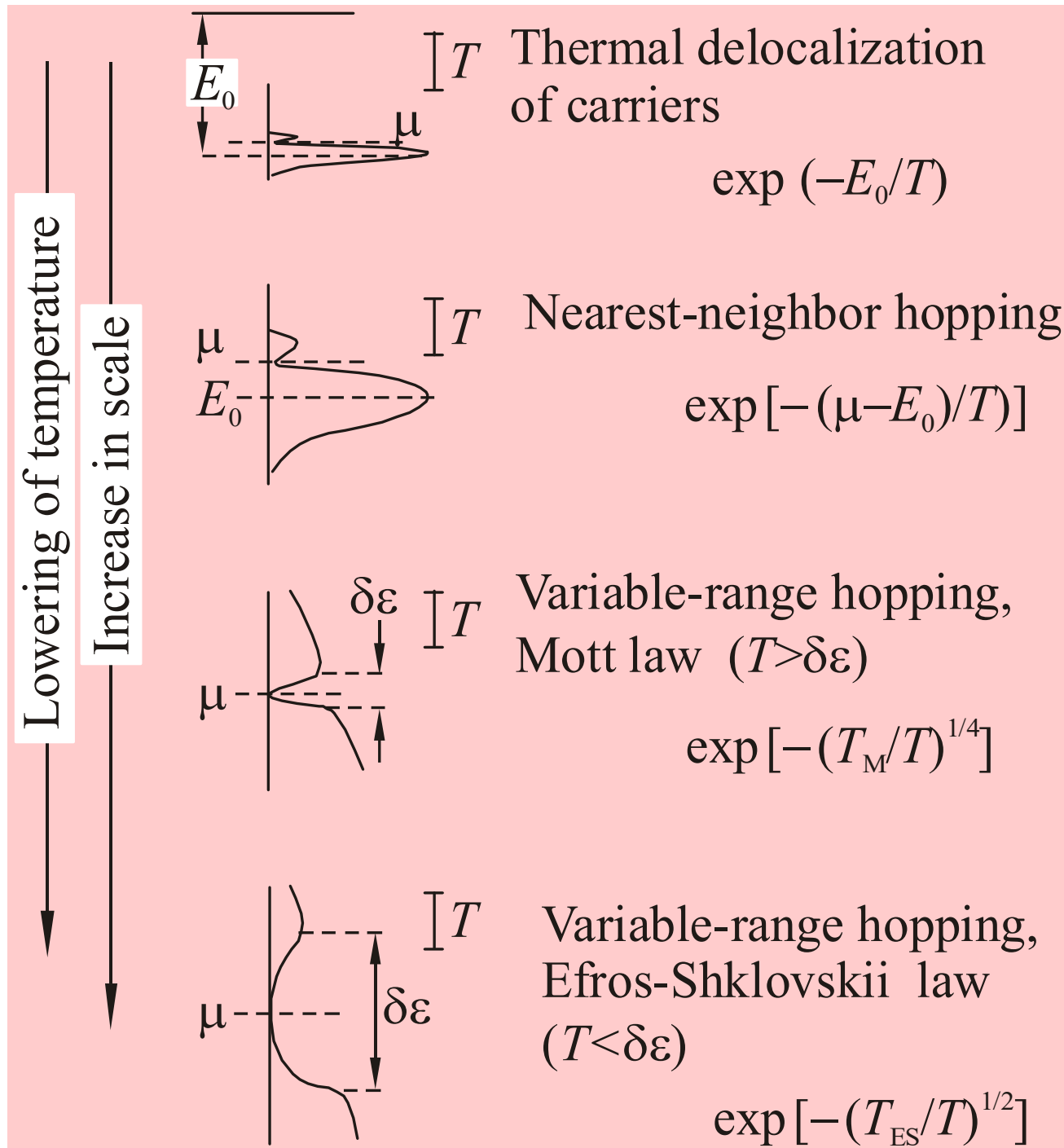
$$r_{ij} = [N(\varepsilon)]^{-1/d} = \frac{e^2}{\kappa \varepsilon}, \quad u_{ij} = \frac{2}{a_B [N(\varepsilon)]^{1/d}} + \frac{\varepsilon}{T} = \frac{2e^2}{\kappa a_B \varepsilon} + \frac{\varepsilon}{T},$$

$$\varepsilon_{\min} = \left(\frac{2e^2 T}{\kappa a_B} \right)^{1/2} = (TT_{ES})^{1/2}, \quad T_{ES} = \frac{2e^2}{\kappa a_B}$$

resistance

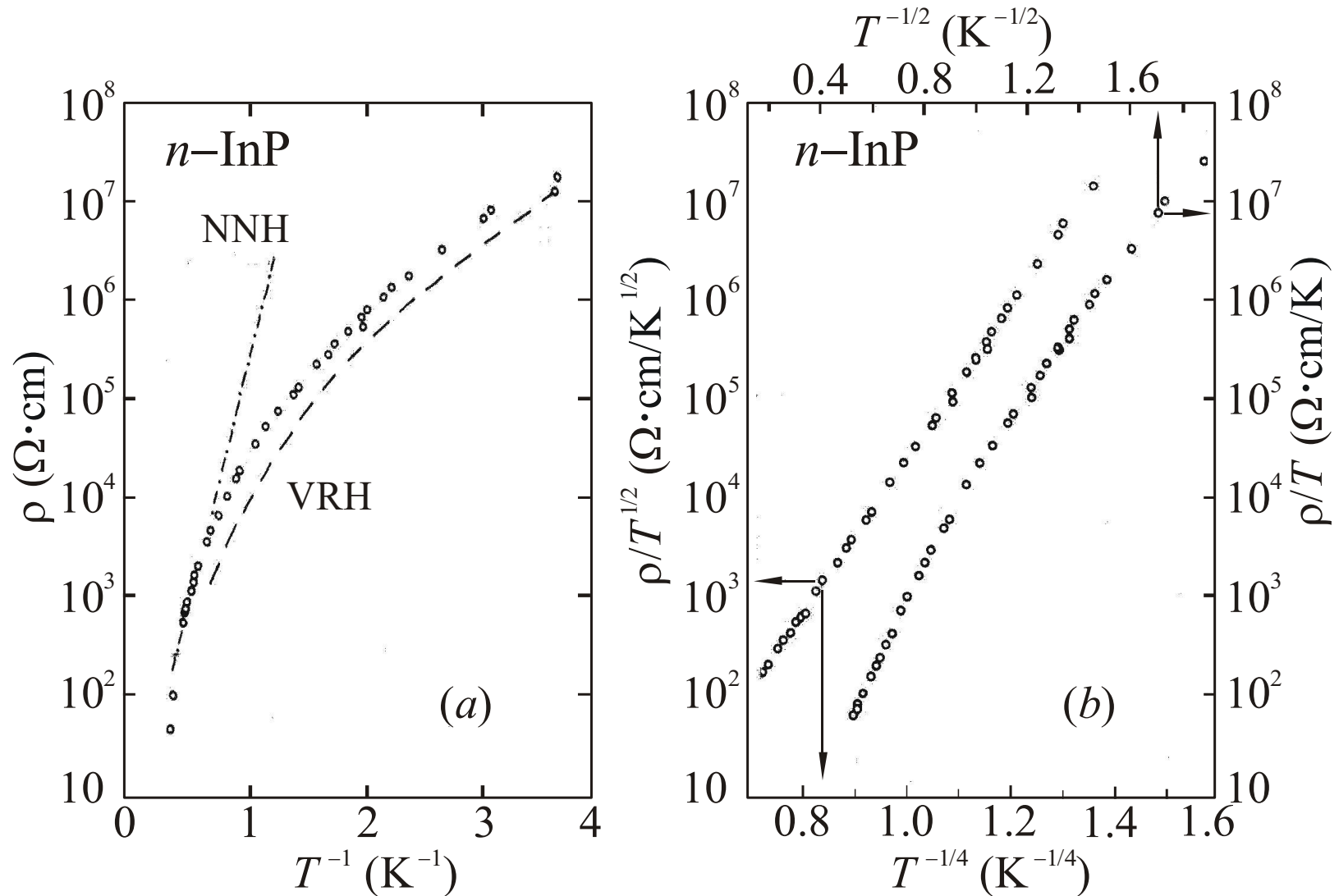
$$\rho = \rho_0 \exp \left(\frac{T_{ES}}{T} \right)^{1/2} \quad (d = 3, 2)$$

Mechanisms of hopping conductivity



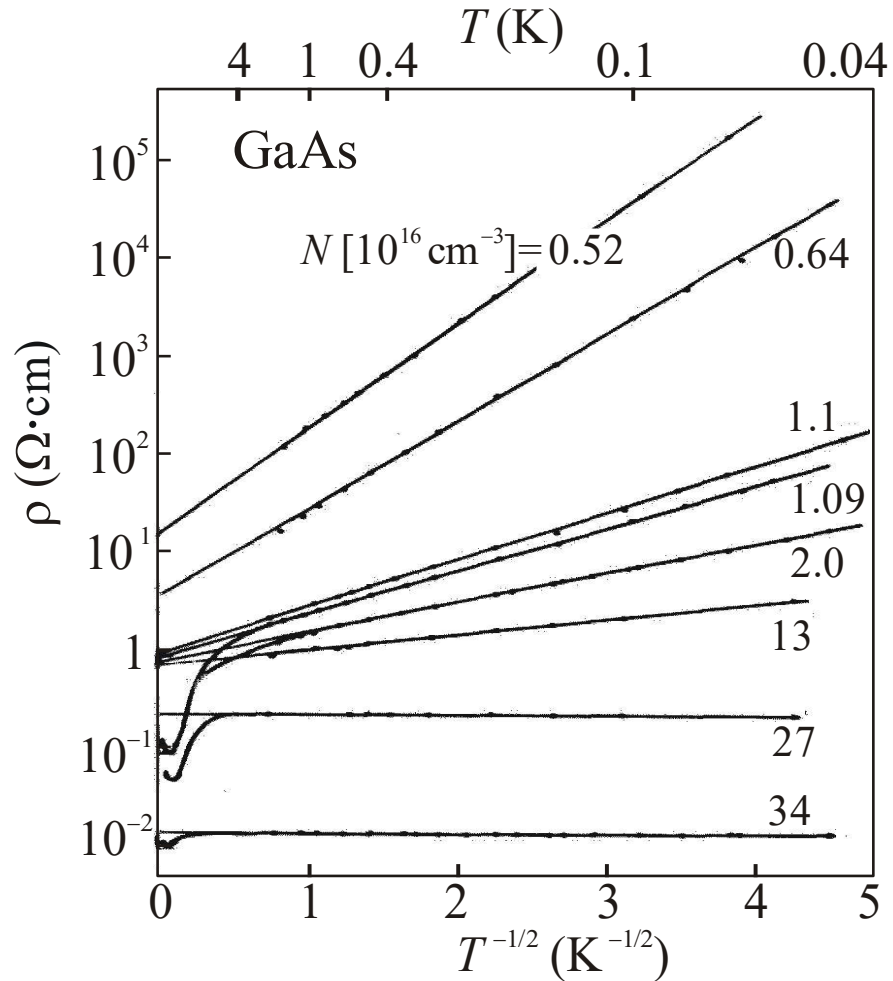
Variable range hopping: experiment

temperature dependence, fitting by standard functions

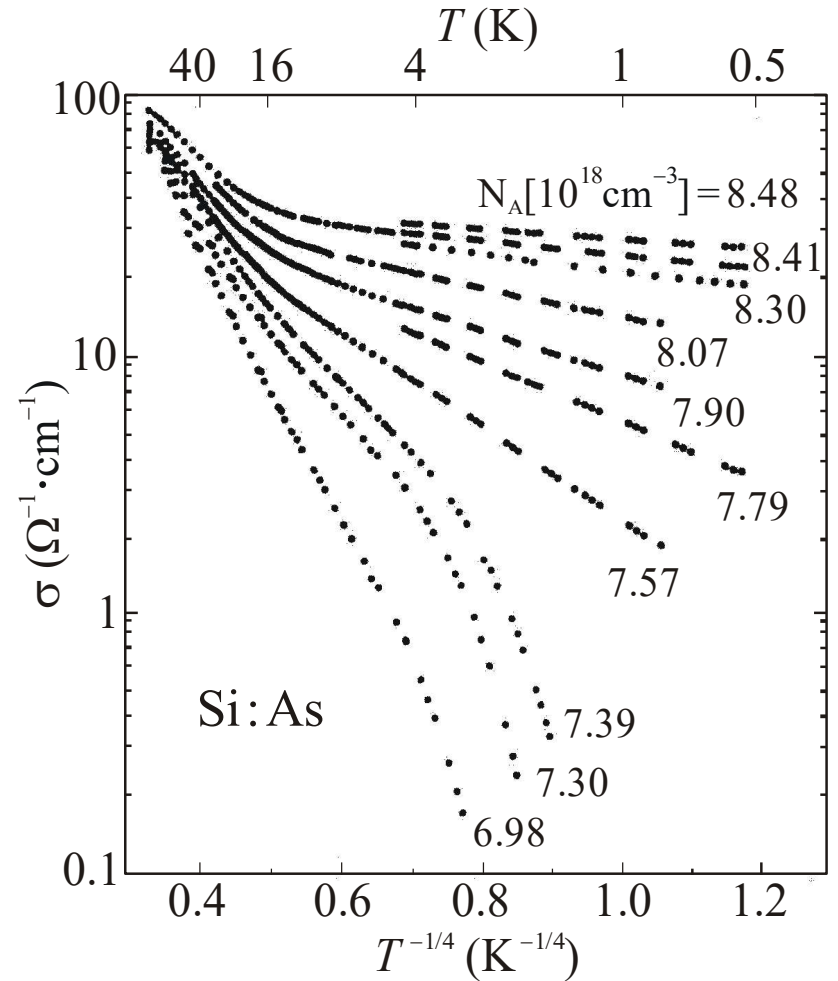


R. Mansfield, S. Abboudy, F. Foozoni, Philos.Mag. B **57**, 777 (1988)

experiments

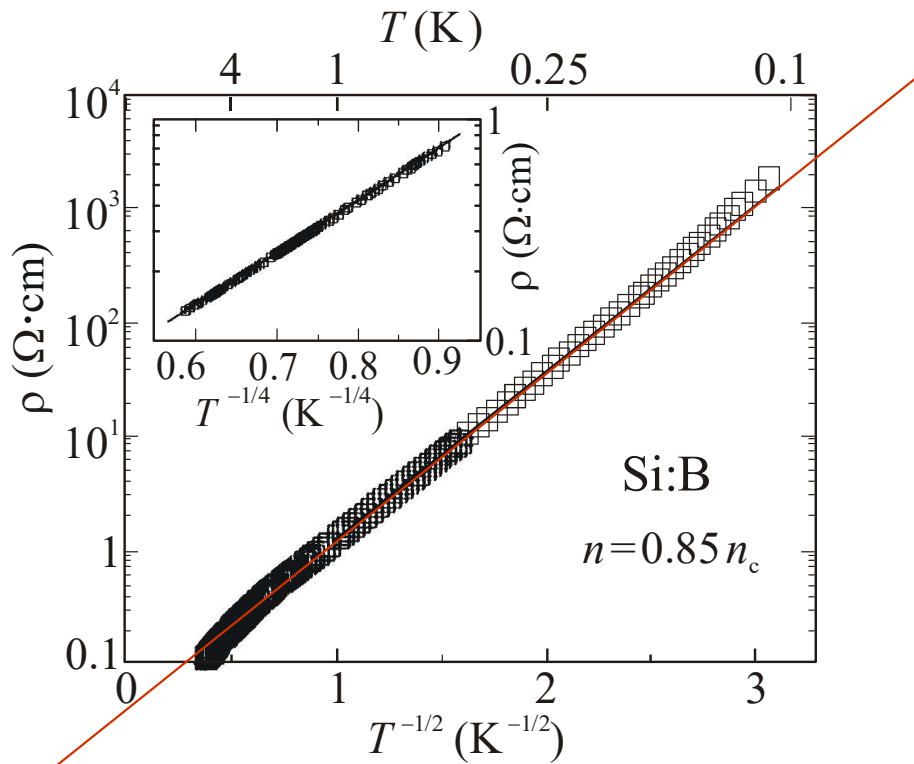


R. Rentsch, K.J. Friedland, A.N. Ionov,
et al.,
phys. stat. solidi b **137**, 691 (1986)



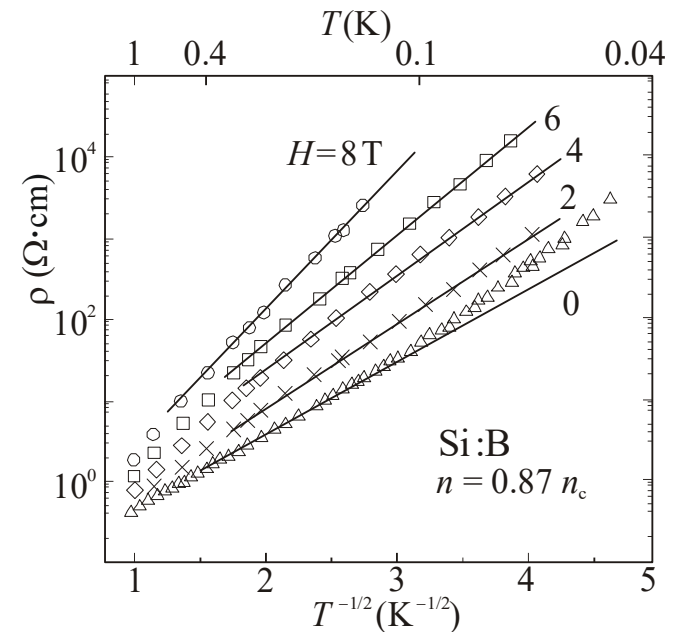
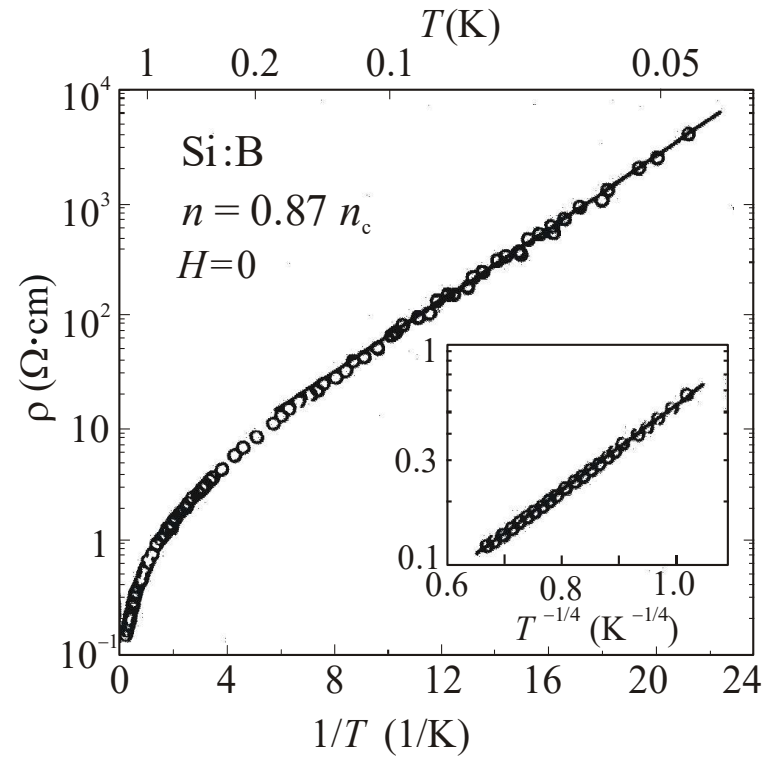
W.N. Shafarman, D.W.Koon,
T.G. Castner,
PRB **40**, 1216 (1989)

More experiments: *Si:B*



J.G. Massey, M. Lee,
PRL **75**, 4266 (1995)

P. Dai, Y. Zhang, M.P. Sarachik, PRL **69**, 1804 (1992)



Electric-field activated variable-range hopping transport in $\text{PrBa}_2\text{Cu}_3\text{O}_{7-\delta}$

G. K. van Ancum, M. A. J. Verhoeven, D. H. A. Blank, and H. Rogalla

Department of Applied Physics, University of Twente, P.O. Box 217, NL-7500 AE Enschede, The Netherlands

(Received 25 January 1995)

We demonstrate the transport of charge carriers in $\text{PrBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (PBCO) to be dependent both on the applied electric field and on the temperature. In our measurements we use inert noble-metal contacts on laser ablated and sputtered PBCO films. By applying the transmission line model we are able to separate the contact resistance from the PBCO resistance. The average hopping distance can be found by extending Mott's formula to field activation, and is found to be much greater than the dimensions of the PBCO unit cell. From the measurements in strong electric field a minimum hopping distance in the direction of the applied field of about 14 nm is determined, which we discuss in terms of localized states and intrinsic mixed valence of the Pr atoms in the PBCO film.

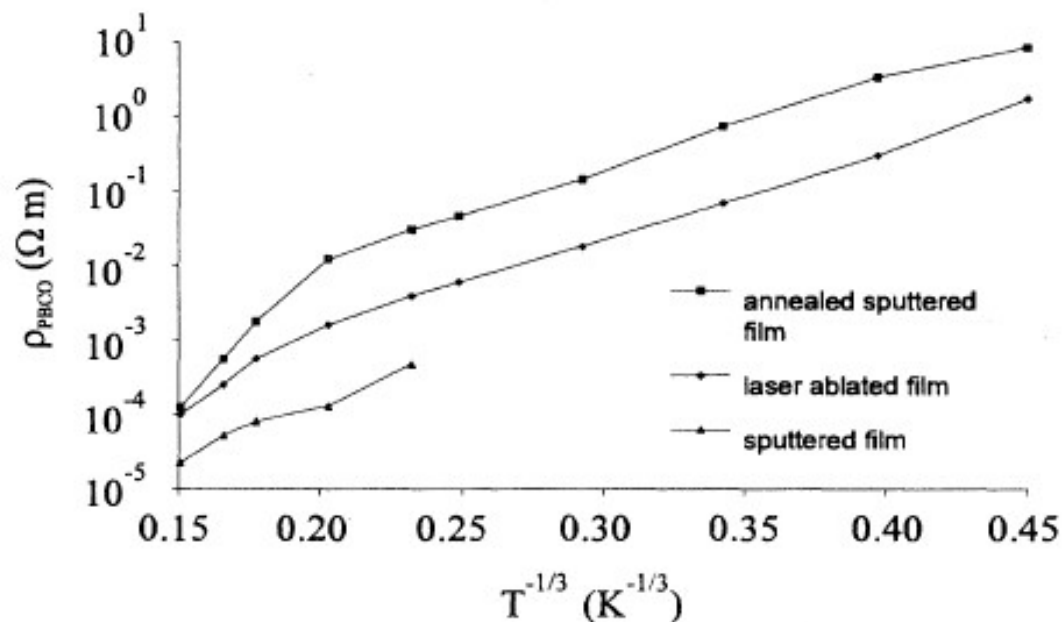


FIG. 3. The PBCO resistivity ρ_{PBCO} for laser ablated and sputtered films in zero electric field.

Magnetoresistance of $\text{PrBa}_2\text{Cu}_3\text{O}_{7-\delta}$ thin films

G. K. van Ancum, M. A. J. Verhoeven, D. H. A. Blank, and H. Rogalla

Department of Applied Physics, University of Twente, P.O. Box 217, NL-7500 AE Enschede, The Netherlands

(Received 26 June 1995)

Transport of charge carriers in $\text{PrBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (PBCO) is often described by variable-range hopping (VRH). Until now the VRH mechanism was confirmed merely on the basis of a temperature dependence of the resistivity following Mott's law. In this article we show a positive magnetoresistance in PBCO thin films, depending exponentially on the applied magnetic field. This provides substantial additional evidence for a variable-range hopping transport mechanism. Both a strong-field and a weak-field magnetoresistance can be identified. At temperatures above 30 K we observe weak-field magnetoresistance, at 4.2 K we detect a transition from weak-field to strong-field magnetoresistance at a magnetic field of approximately 4.5 T. In the weak-field regime the radius of the localized wave function is only affected marginally by the applied magnetic field. In the strong-field regime the radius of the localized wave function decreases with increasing magnetic field. From the measurements in the strong-field regime we obtain an estimate for the two-dimensional density of localized states in the PBCO thin film of approximately $2 \times 10^{13} \text{ 1/eVm}^2$.

$$a_H = a_0 / \ln \left(H \frac{k_B^2 \hbar^3}{m^2 e^3} \right) \quad \text{for } \lambda \leq a_0,$$

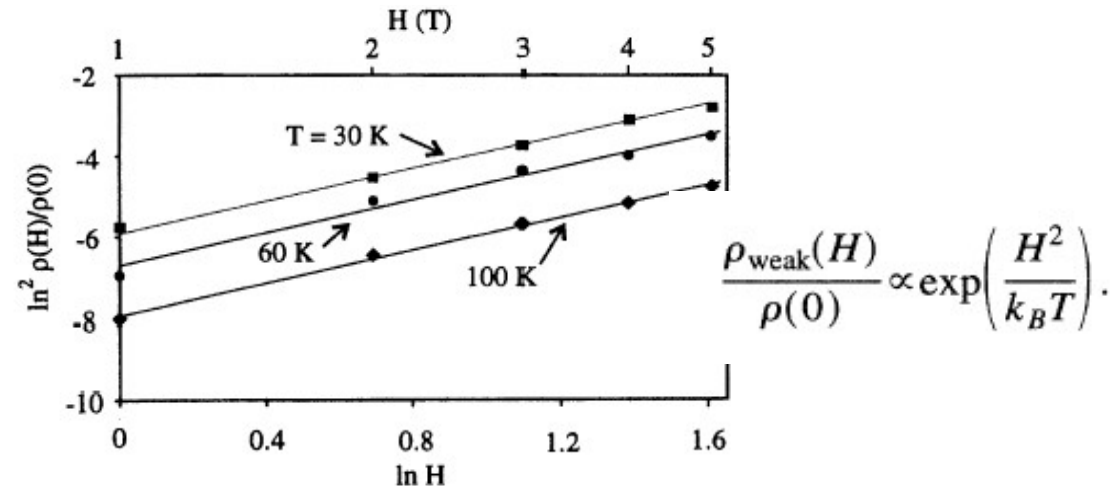
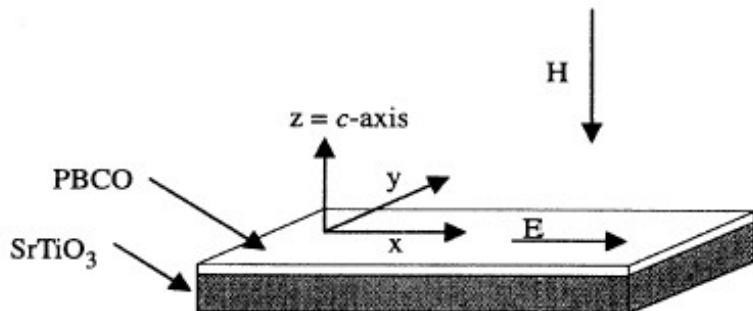


FIG. 2. Magnetoresistance at 100, 60, and 30 K (sample No. 1). The drawn lines represent weak-field dependence.

FIG. 1. Schematic drawing of the measurement setup.