# Quantum error mitigation in the regime of high noise using deep neural network: Trotterized dynamics

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**Abstract.** We address a learning-based quantum error mitigation method, which utilizes deep neural network applied at the postprocessing stage, and study its performance in presence of different types of quantum noises. We concentrate on the simulation of Trotterized dynamics of 2D spin lattice in the regime of high noise, when expectation values of bounded traceless observables are strongly suppressed. By using numerical simulations, we demonstrate a dramatic improvement of data quality for both local weight-1 $\langle Z \rangle$ and weight-2 $\langle ZZ \rangle$ observables for the depolarizing and inhomogeneous Pauli channels. At the same time, the effect of coherent $ZZ$ crosstalks is not mitigated, so that in practise crosstalks should be at first converted into incoherent errors by randomized compiling.

**Keywords:** quantum algorithms, machine learning, neural networks, quantum error mitigation, Ising model, NISQ processors, Trotter evolution

Noisy intermediate-scale quantum (NISQ) devices represent the current edge of quantum computing technology [1]. Particularly, such processors can be useful for solving evolutionary problems. However, the simulation of the dynamics of such systems at long times requires a large number of Trotter decomposition steps of evolution operator. This leads to the fact that a large number of quantum gates are required for simulation, which means that the outcomes from the quantum computer become too noisy. To address these limitations, advanced quantum error mitigation strategies like probabilistic error cancellation and zero noise extrapolation have been developed and are proving critical in enhancing the utility of NISQ machines.

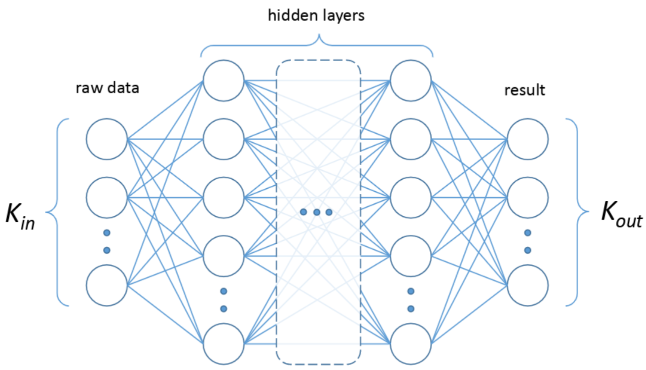


Figure 1: The schematic view of the DNN structure. The number of input (output) neurons is $K_{in}$ ($K_{out}$). Several hidden layers can be used. In our illustrative simulations we used three hidden layers each consisting of 1000 neurons. The sigmoid activation function after both hidden and output layers is utilized, while ReLu activation function is used for hidden layers.

In our earlier work, we proposed a learning-based method to mitigate quantum errors using deep neural networks (DNNs) [2]. This method focuses on optimizing error reduction in quantum circuits, specifically those employed in Trotterized quantum simulations. The fundamental concept involves training a deep neural network (see Fig. 1) with data from shallower, less noisy circuits, and then applying this model to deeper, noisier circuits. In order to get noisy data for training we artificially increase of the quantum circuits depth by incorporation of fictitious Trotter blocks formally equivalent to identity gates into the circuit (see Fig. 2). Their role is to increase noise level due to the hardware imperfections while preserving the circuit's general structure and its relevant features.

The main goal of this study is to thoroughly evaluate the performance of our neural network-based approach for quantum error mitigation under various noise conditions [3]. We focus on conducting detailed numerical simulations to differentiate between distinct types of quantum noises, which is particularly crucial for understanding the complex dynamics in superconducting quantum devices. Our study specifically investigates the impact of several noise channels, including depolarizing and inhomogeneous Pauli noises, as well as ZZ crosstalk, which is particularly challenging in fixed-frequency superconducting qubits. The ultimate goal is to demonstrate marked improvements in data quality for all these noise channels. This research aims to establish a robust method that effectively combines machine learning with quantum error mitigation techniques to significantly improve the accuracy and practicality of NISQ devices.

**Model.** As a test case, we consider the dynamics of 2D spin lattice described by the transverse-field Ising model Hamiltonian

$$H = -\sum_j h_j X_j - \sum_{<ij>} J_{ij} Z_i Z_j, \qquad (1)$$

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where $h_j$ are local transverse fields and $J_{ij}$ are coupling constants which are nonzero only for nearest neighbors and randomly distributed according to the Gaussian distribution.

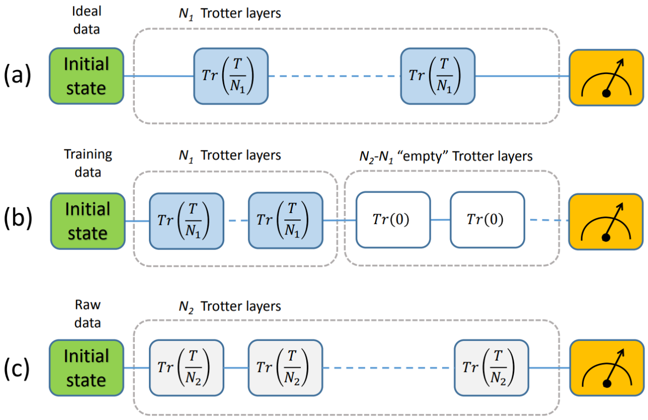


Figure 2: Schematic view of the method. (a) - Generation of ideal or quasi-ideal data using classical computation or the same quantum computer corresponding to $N_1$ Trotter layers and different initial conditions. (b) - DNN training by adding $N_2 - N_1$ "empty" Trotter layers to the quantum circuit and transforming such noisy data towards their ideal counterparts. (c) - applying DNN to the noisy data corresponding to $N_2$ Trotter layers.

The dynamics of the system starting from a given initial state can be simulated digitally using Trotter decomposition of the evolution operator. The total evolution time $T$ can be discretized into $N$ time steps $\delta t = T/N$. The evolution operator for each Trotter layer can be written in a standard way as a product of two operators given by

$$e^{-iH_{ZZ}\delta t} = \prod_{<ij>} R_{Z_i Z_j}(2J_{ij}\delta t), \qquad (2)$$
$$e^{-iH_X \delta t} = \prod_i R_{X_i}(2h_i \delta t),$$

where $R_{Z_i Z_j}$ and $R_{X_i}$ are $ZZ$ and $X$ rotation gates, respectively.

**Results.** As demonstration, we consider weight-1 observables, such as individual magnetizations of spins in $z$ direction, $\langle Z_j \rangle$. We perform simulations for the 2D square lattice containing $n = 9$ spins.

In Fig. 3 we show the mean magnetization $\langle Z \rangle = 1/n \sum_j \langle Z_j \rangle$ for a particular realization of disorder as a function of time for Trotter step numbers $N_2 = 32$ (a), $N_2 = 64$ (b).

We see a dramatic improvement of the data quality by the DNN despite of the fact that the raw observables are strongly suppressed by the noise and for very large $N_2$ tend to cluster around 0. The quality of error mitigation for very large $N_2$ becomes limited essentially by the statistic uncertainties associated with the probabilistic nature of measurement (shot noise). DNN naturally amplifies the shot noise, since it also amplifies the whole signal. However, the shot noise produces no bias, in contrast to quantum gate errors, which do produce bias. Note that the shot noise on mitigated curves can be additionally smoothed by using some other technique.

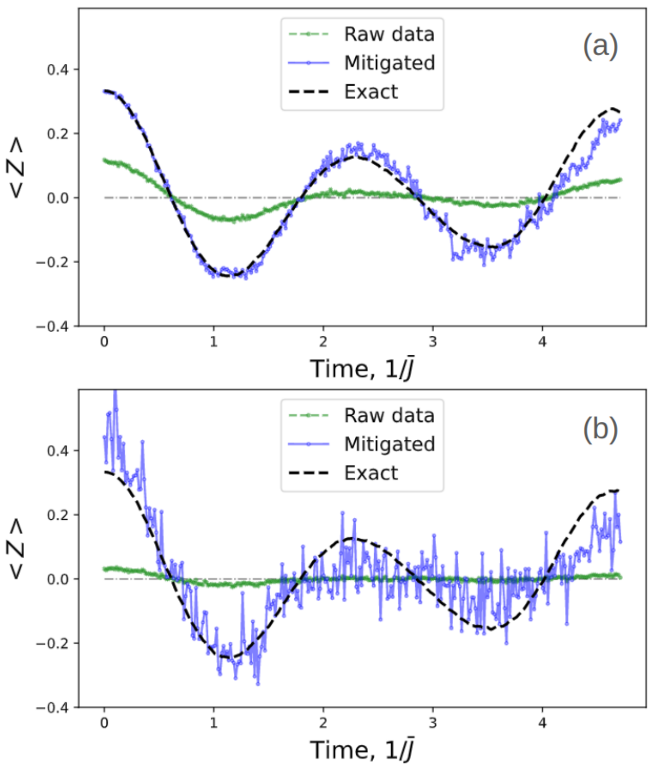


Figure 3: The dependence of a mean magnetization in $z$ direction $\langle Z \rangle$ of 9-spin system on time for Trotter layer numbers $N_2 = 32$ (a), $N_2 = 64$ (b) starting from the initial condition $|000111000\rangle$ at $\bar{h} = 2\bar{J}$.

**Conclusion.** In this study, we evaluated the effectiveness of a DNN-based quantum error mitigation method. Our results showed significant improvements in data quality for incoherent noises, such as depolarizing and inhomogeneous Pauli channels. However, coherent errors due to ZZ crosstalks are not mitigated. We concluded that such noises should be converted into incoherent errors by randomized compiling before using of our method.

## References


[1] J. Preskill. Quantum computing in the NISQ era and beyond. In *Quantum* 2:79, 2018.

[2] A.A. Zhukov, W.V. Pogosov. Quantum error reduction with deep neural network applied at the post-processing stage. In *Quant. Inf. Proc.* 21:93, 2022.

[3] A.A. Zhukov, W.V. Pogosov. Quantum error mitigation in the regime of high noise using deep neural network: Trotterized dynamics. In *Quant. Inf. Proc.* 23:80, 2024.