Wave mixing of classical and nonclassical light on a single qubit

W. V. Pogosov[1,2,3],

[1] Research Institute of Automatics (VNIIA), Moscow,

[2] Moscow Institute for Physics and Technology, Dolgoprudny,

[3] Institute for Theoretical and Applied Electrodynamics RAS, Moscow

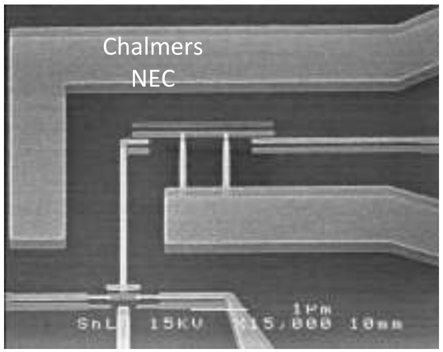
In collaboration with:

A. Yu. Dmitriev, A. V. Vasenin, S. A. Gunin, O. V. Astafiev (MIPT, Skoltech);

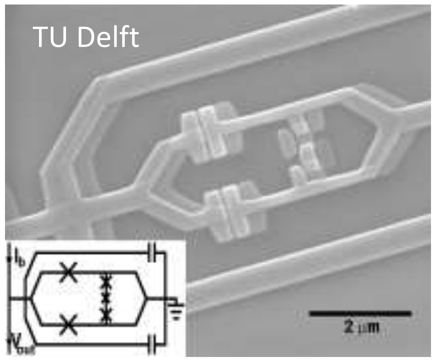
S. V. Remizov, A. A. Elistratov (VNIIA)

Superconducting qubits

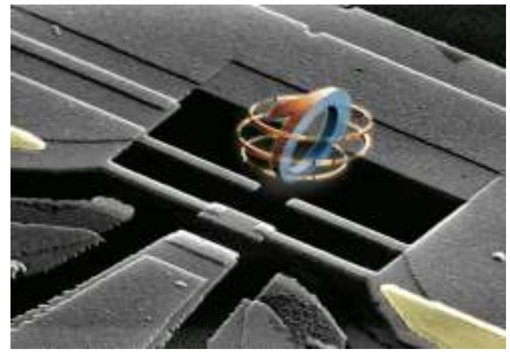
charge 1999



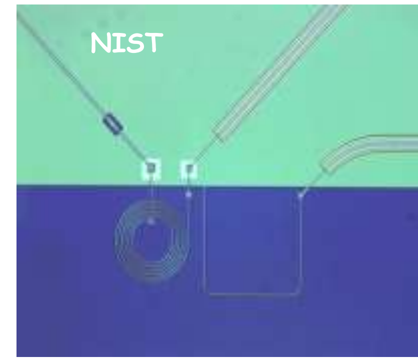
flux 2002

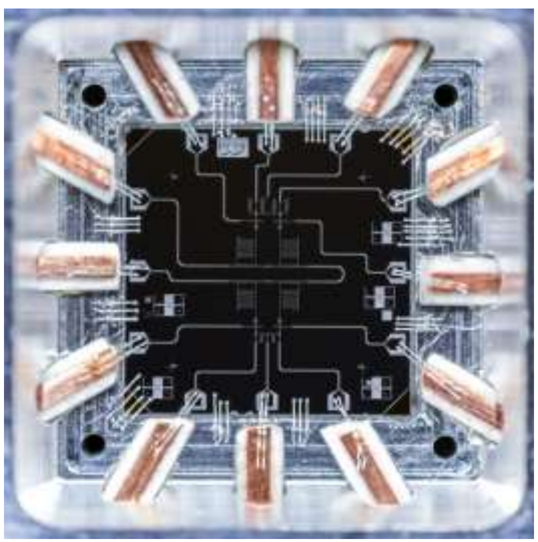


charge-/flux 2004

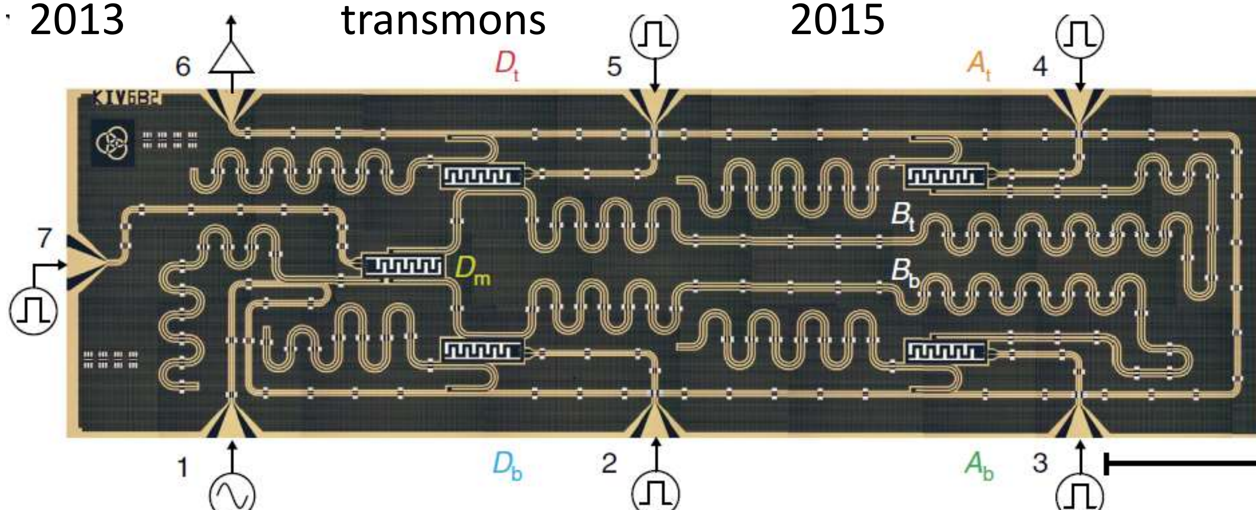


phase 2004





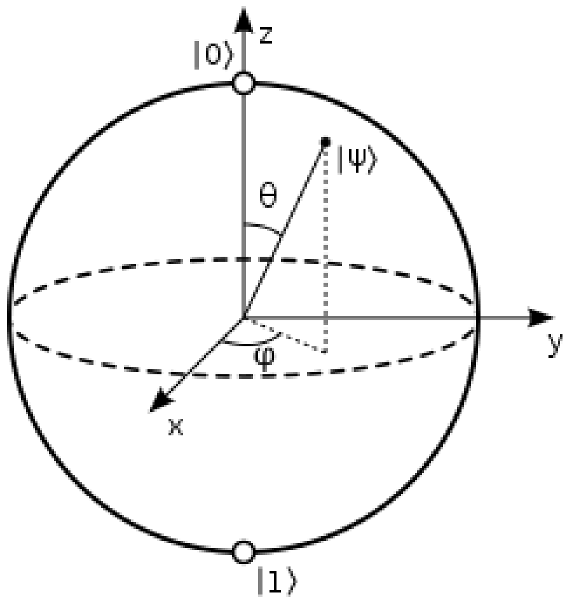
2013 transmons 2015



Кубит (двухуровневая система)

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Сфера Блоха



$$\langle\sigma_z\rangle = \cos\theta$$

$$\langle\sigma_-\rangle = \frac{1}{2}e^{i\phi}\sin\theta$$

Superconducting qubits as nonlinear elements for the photon interaction

A series of works by the group(s) of O. V. Astafiev

Nat. Commun. 8, 1352 (2017); Phys. Rev. A 98, 041801(R) (2018); Phys. Rev. A 100, 013808 (2019),...

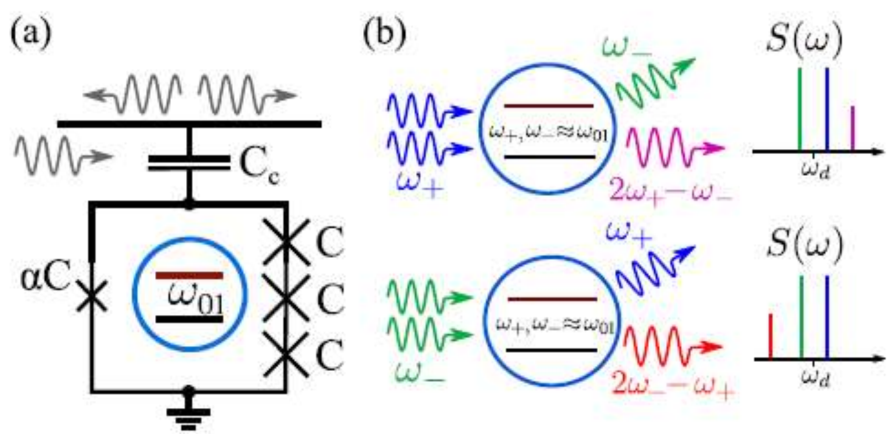
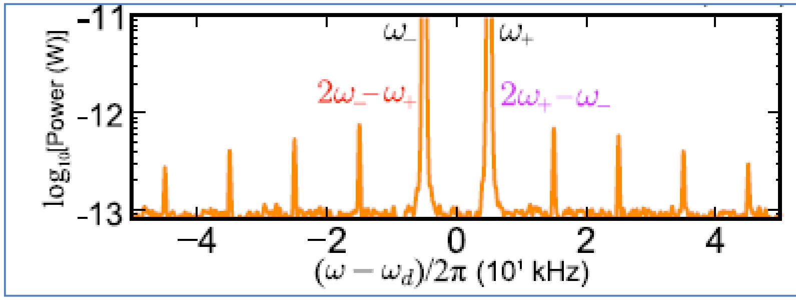


FIG. 1. (a) Schematics of the device. The artificial atom is a 4-junction flux qubit with three nearly identical junctions with capacitance C and one junction with capacitance αC, where $\alpha = 0.43$. The qubit is coupled to a coplanar waveguide with capacitance C_c. (b) The wave mixing of two tones on a single two-level system. With the two-photon absorption and one-photon emission at frequencies $\omega_\pm$ and $\omega_\mp$, one more photon is generated at frequency $2\omega_\pm - \omega_\mp$, resulting in corresponding side frequency emissions.

Spectrum of scattered radiation (spectrum analyzer):



Amplitude of the elastically scattered wave:

$$V^{sc} = -\frac{i\Gamma_1}{\hbar\mu}\langle\sigma^-\rangle$$

$$\boxed{\langle\sigma^-\rangle = -\frac{\sin\theta}{\Lambda}\frac{\Omega_- e^{-i\delta\omega t} + \Omega_+ e^{i\delta\omega t}}{1 + \sin\theta\cos 2\delta\omega t}}$$

Hypothesis: peak heights should be sensitive to photon statistics

I. Two classical signals

Closed frequencies: $|\omega_1 - \omega_2| \ll \Gamma$.

Amplitudes of drivings: Ω_1 и Ω_2.

Rotating frame: $\omega_d = (\omega_1 + \omega_2)/2$

Maxwell-Bloch equations in the rotating-wave approximation:

$$\frac{d\langle\sigma_-\rangle}{dt} = \langle\sigma_-\rangle\left(-i\Delta\omega - \gamma\right) - \frac{i\Omega_1}{2}e^{-i\delta\omega t}\langle\sigma_z\rangle - \frac{i\Omega_2}{2}e^{i\delta\omega t}\langle\sigma_z\rangle,$$

$$\frac{d\langle\sigma_z\rangle}{dt} = -\Gamma(\langle\sigma_z\rangle + 1) + i\Omega_1\left(\langle\sigma_+\rangle e^{-i\delta\omega t} - \langle\sigma_-\rangle e^{i\delta\omega t}\right) + i\Omega_2\left(\langle\sigma_+\rangle e^{i\delta\omega t} - \langle\sigma_-\rangle e^{-i\delta\omega t}\right),$$

where $\Delta\omega = \omega_{01} - \omega_d$, $\delta\omega = \omega_1 - \omega_d = \omega_d - \omega_2$.

Γ is a radiative decay rate

$\gamma = \Gamma/2 + \Gamma_\phi$ is a decoherence rate

Γ_ϕ is a pure dephasing rate

I. Two classical signals

$\delta\omega t$ is a slowly varying phase on the time scale Γ^{-1}

Quasi-stationary solutions of Maxwell-Bloch equations

$$\langle\sigma_z\rangle = -\left(1 + \frac{\gamma}{\Gamma}\frac{\Omega_1^2 + \Omega_2^2 + \Omega_1\Omega_2(e^{-2i\delta\omega t} + e^{2i\delta\omega t})}{(\Delta\omega)^2 + \gamma^2}\right)^{-1},$$

$$\langle\sigma_-\rangle = \frac{1}{2}\frac{\Omega_1 e^{-i\delta\omega t} + \Omega_2 e^{i\delta\omega t}}{\Delta\omega - i\gamma}\left(1 + \frac{\gamma}{\Gamma}\frac{\Omega_1^2 + \Omega_2^2 + \Omega_1\Omega_2(e^{-2i\delta\omega t} + e^{2i\delta\omega t})}{(\Delta\omega)^2 + \gamma^2}\right)^{-1}.$$

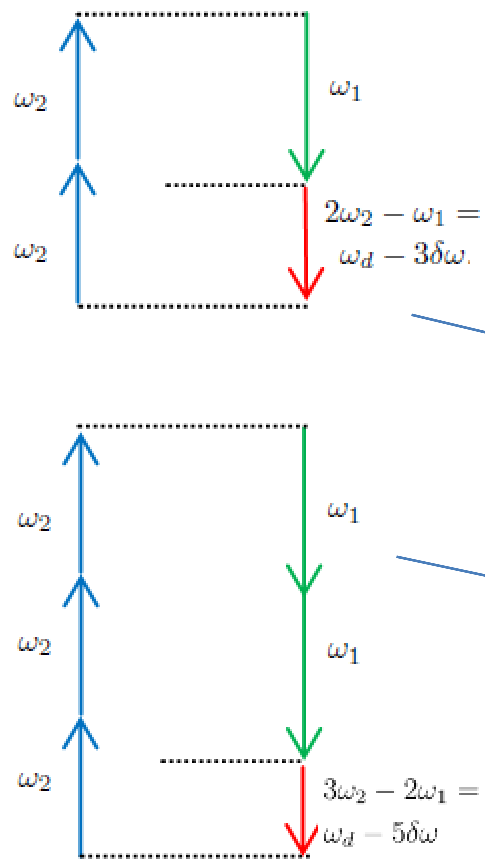
Amplitude of the scattered wave is $-i\Gamma\langle\sigma_-\rangle/\mu$,

where μ is the qubit dipole moment

I. Two classical signals

$$\langle\sigma_-\rangle = \frac{\Omega_1 e^{-i\delta\omega t} + \Omega_2 e^{i\delta\omega t}}{\Lambda}\tan\vartheta \sum_{p=-\infty}^{+\infty}(-\tan(\vartheta/2))^{|p|}e^{i2p\delta\omega t},$$

$$\Lambda = \frac{4\gamma\Omega_1\Omega_2}{\Gamma(\Delta\omega + i\gamma)}, \qquad \vartheta = \arcsin\frac{2\gamma\Omega_1\Omega_2}{\Gamma\left((\Delta\omega)^2+\gamma^2\right)+\gamma(\Omega_1^2+\Omega_2^2)}.$$



Spectral components

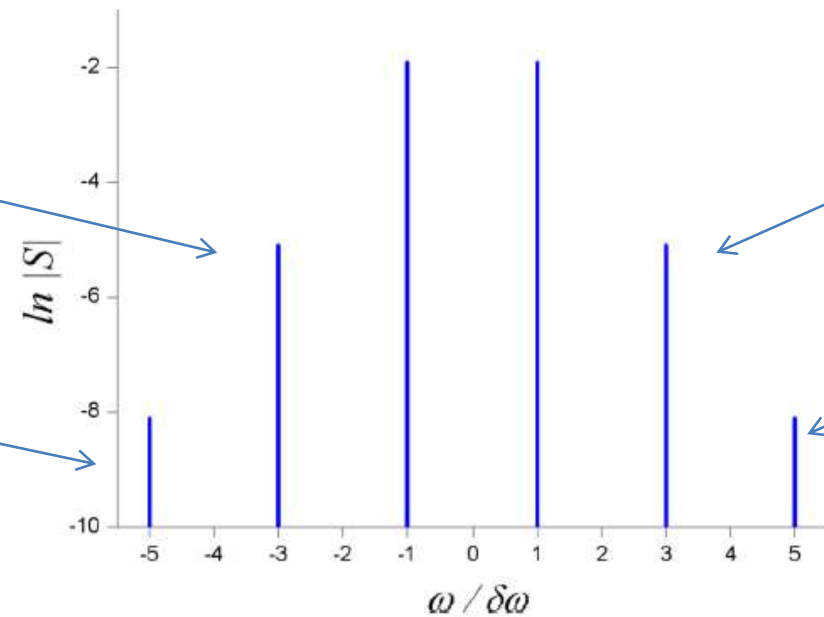
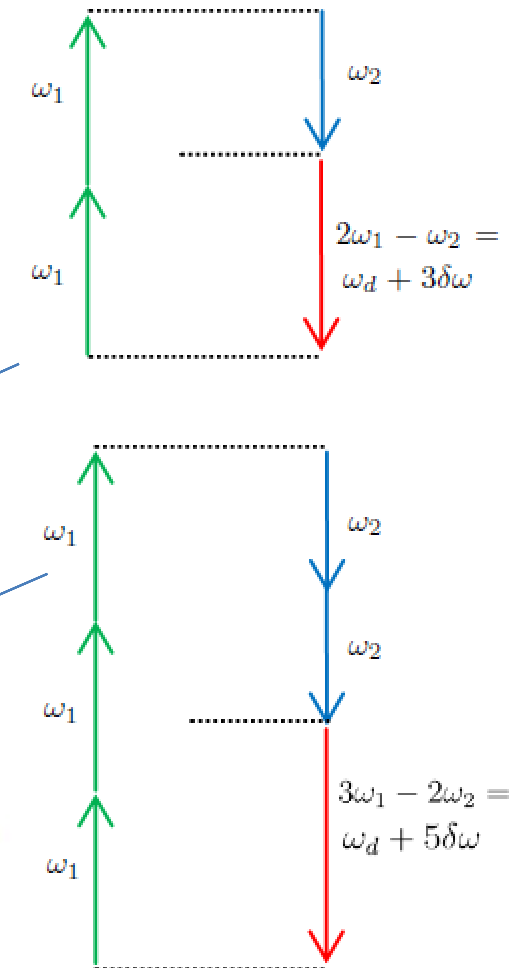


FIG. 1. Spectral components of $\langle\sigma_-\rangle$ in the case of qubit irradiation by two coherent waves (see text).



$$\Omega_1 = \Omega_2 = 0.15\Gamma,\ \Delta\omega = 0,\ \Gamma = 2\gamma.$$

I. Two classical signals

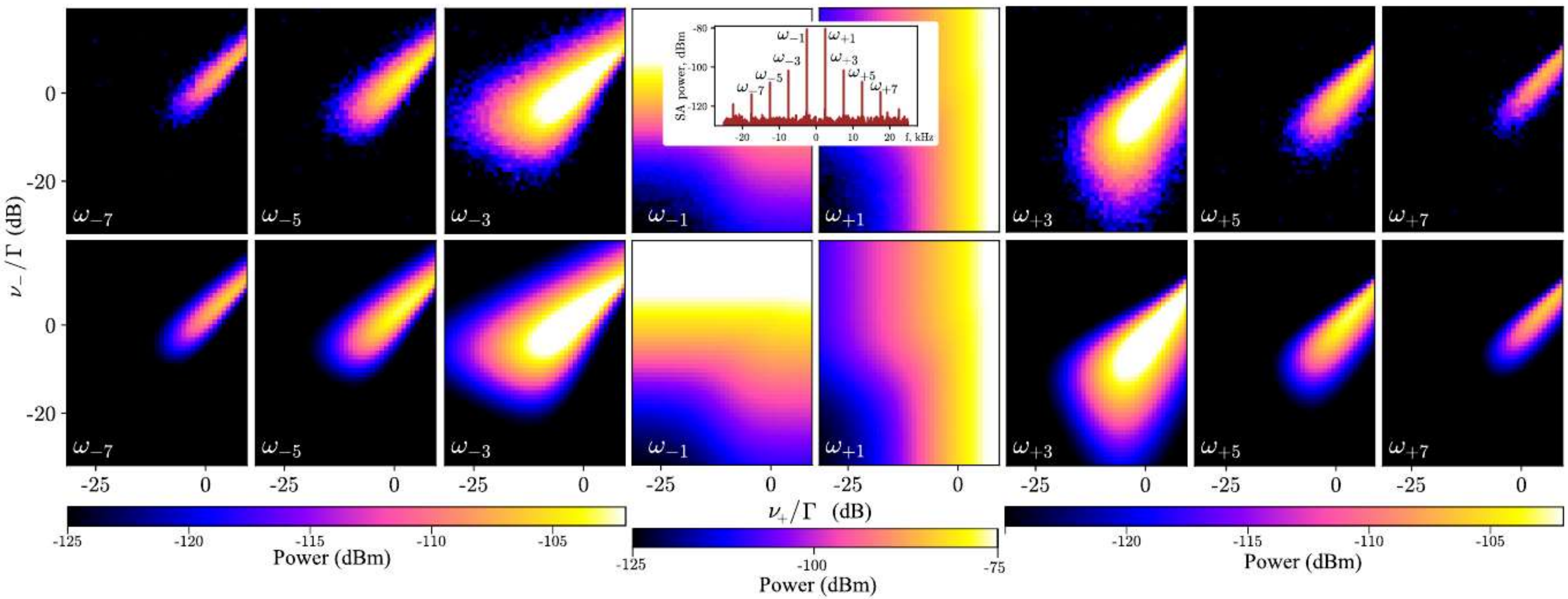
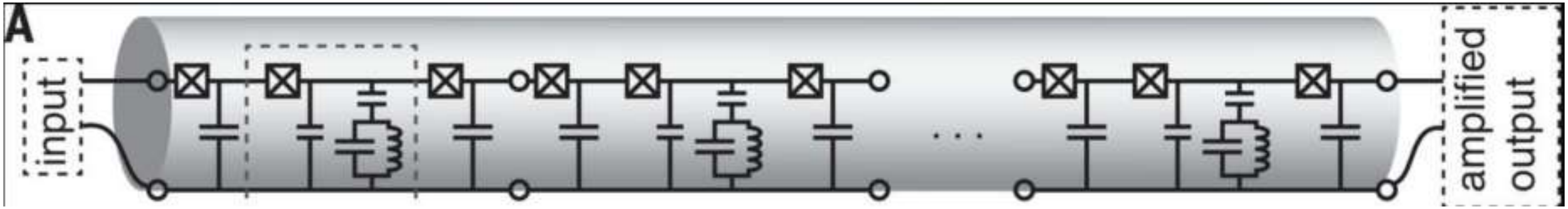


FIG. 4. The mixing of two classical tones of frequencies ω_+ and ω_- on the probe. The tone of ω_+ is applied through the input `in I` when the source is very detuned and its effect is negligible, see Fig. 1(b), and the tone of ω_- is applied through the input `in II`. The measured powers of the side peaks are plotted in the upper row of the panels as functions of ν_-/Γ and ν_+/Γ. The inset panel in between components ω_{-1} and ω_{+1} of the upper row contains the single-run measured spectrum of mixing for $\nu_-/\Gamma = \nu_+/\Gamma = 0$ dB. The bottom panels depict analytical expressions of Eq. (7) recalculated to the power of the side peaks for $\Gamma = 2$ MHz. All components are re-scaled by a single multiplier to fit the data.

II. Classical signal and squeezed light

Degenerate parametric amplifier (nonlinear medium)



Effective Hamiltonian:

$$H_s = \omega_2 a_0^\dagger a_0 + \frac{i}{2}\left(a_0^{\dagger 2}\epsilon_s e^{-2i\omega_2 t} - a_0^2 \epsilon_s^* e^{2i\omega_2 t}\right)$$

Output field (squeezed light, photon pairs, broadband signal):

$$\langle a_{\text{out}}^\dagger(t) a_{\text{out}}(t')\rangle = N e^{i\omega_2(t-t')}\delta(t-t'),$$

$$\langle a_{\text{out}}(t) a_{\text{out}}(t')\rangle = M e^{-i\omega_2(t+t')}\delta(t-t')$$

II. Classical signal and squeezed light

Equations of motion for the qubit degrees of freedom:

$$\frac{d\langle\sigma_-\rangle}{dt} = \langle\sigma_-\rangle\left(-i\omega_{01} - \gamma(1+2N)\right) - \frac{i\Omega_1}{2}e^{-i\omega_1 t}\langle\sigma_z\rangle - 2\gamma M e^{-2i\omega_2 t}\langle\sigma_+\rangle,$$

$$\frac{d\langle\sigma_z\rangle}{dt} = -\Gamma(\langle\sigma_z\rangle + 1) - 2N\Gamma\sigma_z + i\Omega_1\left(\langle\sigma_+\rangle e^{-i\omega_{d_1}t} - \langle\sigma_-\rangle e^{i\omega_{d_1}t}\right).$$

C. W. Gardiner, *Quantum Noise* (Springer-Verlag, Berlin, 1991).

Quasi-stationary solution:

$$\langle\sigma_z\rangle = -\frac{1}{1+2N} + \frac{\gamma\Omega_1^2}{\Gamma(1+2N)}\frac{1+\frac{M}{2N+1}e^{4i\delta\omega t}+\frac{M^*}{2N+1}e^{-4i\delta\omega t}}{(\Delta\omega)^2+\gamma^2[(2N+1)^2-4|M|^2]+\frac{\gamma\Omega_1^2}{\Gamma}\left(1+\frac{M}{2N+1}e^{4i\delta\omega t}+\frac{M^*}{2N+1}e^{-4i\delta\omega t}\right)},$$

$$\langle\sigma_-\rangle = \frac{\Omega_1}{2}\frac{(i\gamma+\frac{\Delta\omega}{2N+1})e^{-i\delta\omega t}+i\gamma\frac{2M}{2N+1}e^{3i\delta\omega t}}{(\Delta\omega)^2+\gamma^2[(2N+1)^2-4|M|^2]+\frac{\gamma\Omega_1^2}{\Gamma}\left(1+\frac{M}{2N+1}e^{4i\delta\omega t}+\frac{M^*}{2N+1}e^{-4i\delta\omega t}\right)}.$$

II. Classical signal and squeezed light

Spectrum structure

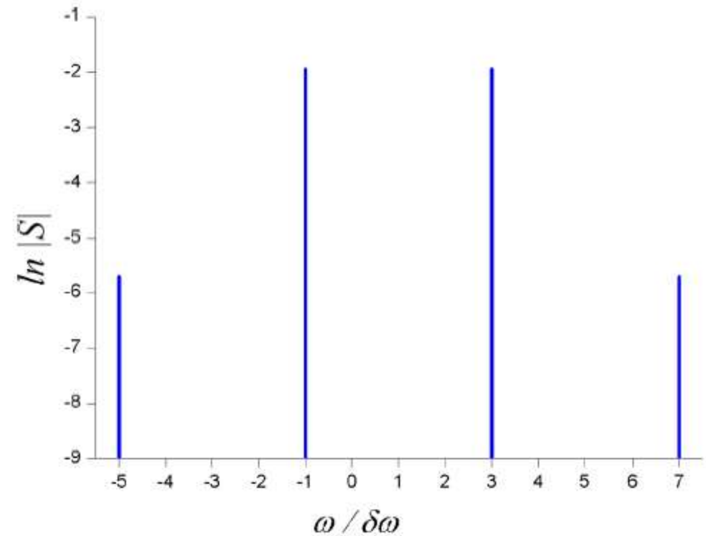


FIG. 2. Spectral components of $\langle\sigma_-\rangle$ in the case of qubit irradiation by a coherent wave together with the pure squeezed light (see text).

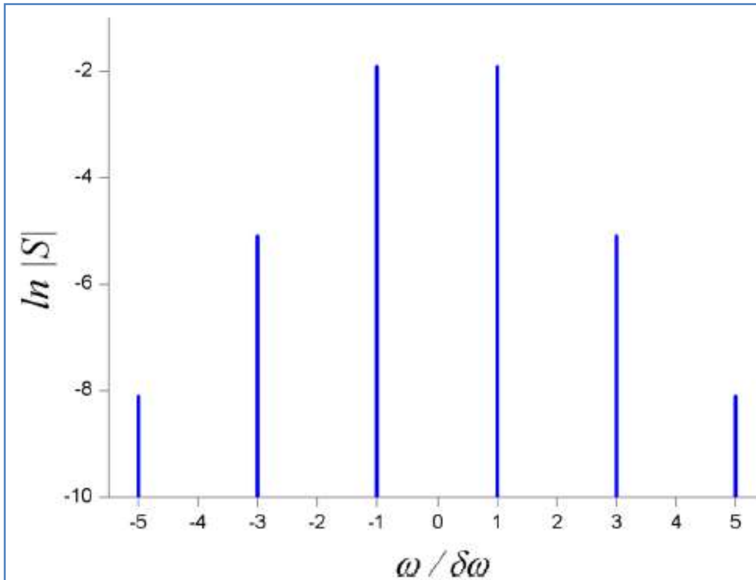


FIG. 1. Spectral components of $\langle\sigma_-\rangle$ in the case of qubit irradiation by two coherent waves (see text).

<u>*Qualitative picture:*</u>

- *The peak structure differs from the case of mixing two classical signals.*
- *Nonzero squeezing (parameter M) together with the classical signal generates side peaks.*
- *Without the classical signal, squeezed light does not generate any peaks.*

The limit of weak driving:

$$\langle\sigma_-\rangle \approx \frac{if\Gamma}{\Omega_1}\left(fe^{-i\delta\omega t} + fme^{3i\delta\omega t} - f^3m^*e^{-5i\delta\omega t} - f^3m^2e^{7i\delta\omega t} + \cdots\right),$$

$$\Omega_1 \ll \Gamma$$

$$f = \frac{\Omega_1}{\sqrt{2\Gamma\gamma[(2N+1)^2 - 4|M|^2]}},$$

$$m = \frac{2M}{2N+1}.$$

Features of multiphoton processes:

- Photons from the squeezed light signal always appear in multiphoton processes as correlated *pairs;* the signal is broadband.

- The amplitudes of the side peaks are proportional to the corresponding powers of the squeezing parameter *m*.

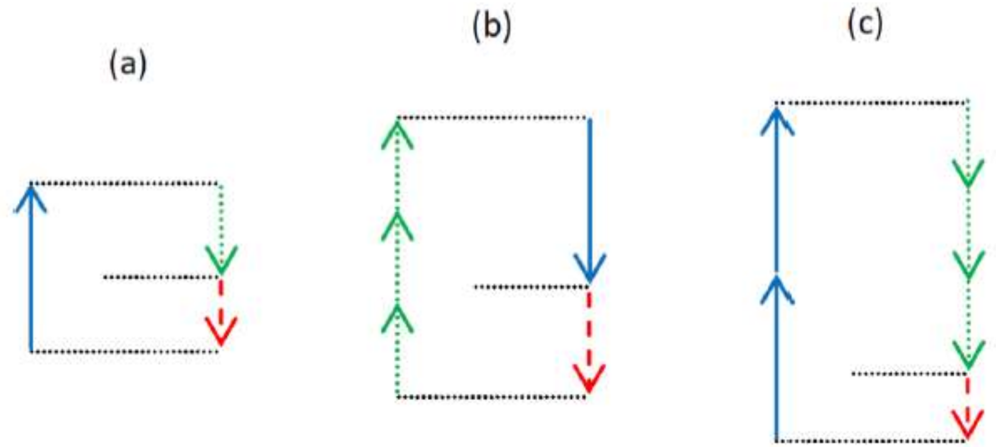


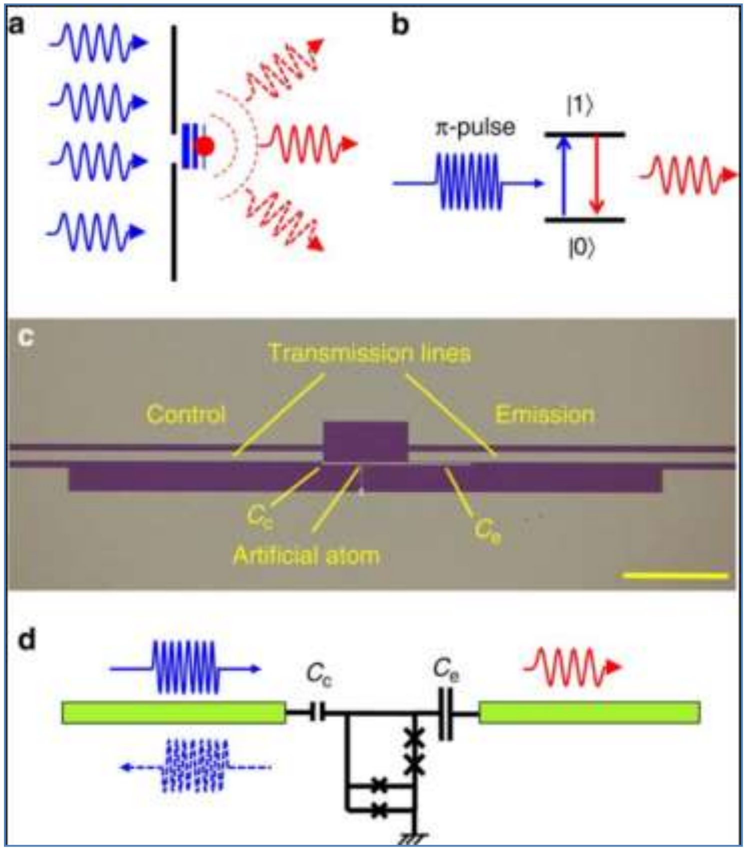
FIG. 3. Schematic images of multiphoton processes resulting in different side peaks in the emission spectra at (a) $3\delta\omega$, (b) $-5\delta\omega$, and (c) $7\delta\omega$. Blue arrows (solid lines) show the absorption and emission of correlated photon pairs with total energy $2\omega_2$. Green arrows (dotted lines) correspond to single photons with energies ω_1. Red arrows (dashed lines) indicate photon emission, which is responsible for the side peaks.

III. Классический сигнал и источник состояний «0 + 1»

Tuneable on-demand single-photon source in the microwave range

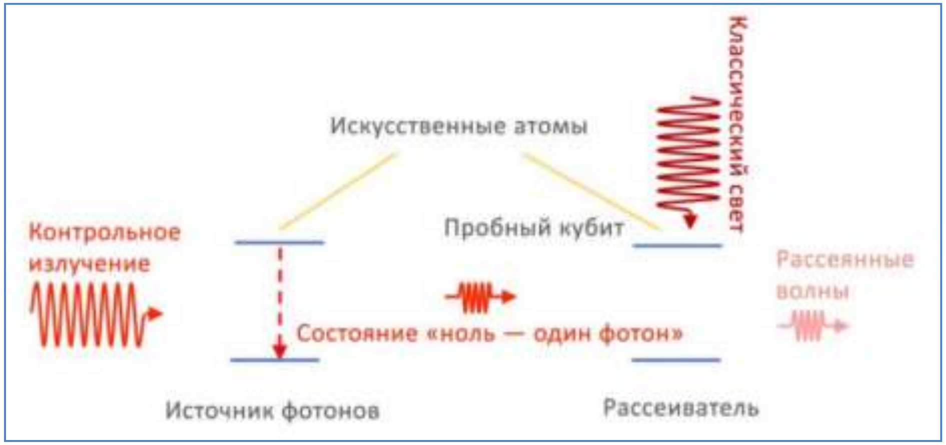
Z. H. Peng, S. E. de Graaf, J. S. Tsai & O. V. Astafiev

Nature Communications **7**, Article number: 12588 (2016)



Смешение волн:

- Что будет, если смешивать классический сигнал и периодически возбуждаемые состояния типа «0+1» с однофотонного источника?



III. Классический сигнал и источник состояний «0 + 1»

Гамильтониан:

$$H = H_{phot} + H_q + H_{int} + H_e + H_{int}^{(e)},$$

$$H_{phot} = \sum_k \omega_k a_k^\dagger a_k,$$

$$H_q = \frac{\omega_{01}}{2}\sigma_z - f_q(t)(\sigma_+ + \sigma_-). \qquad H_{int} = \frac{1}{\sqrt{N_s}}\sum_k (e^{-ikr} g_k^* a_k^\dagger \sigma_- + e^{ikr} g_k a_k \sigma_+)$$

$$H_e = \frac{\omega_{01}^{(e)}}{2}\sigma_z^{(e)} \qquad H_{int}^{(e)} = \frac{1}{\sqrt{N_s}}\sum_k (e^{-ikr_e} g_k^{(e)*} a_k^\dagger \sigma_-^{(e)} + e^{ikr_e} g_k^{(e)} a_k \sigma_+^{(e)})$$

Уравнения движения в гейзенберговской картине:

$$\frac{d\langle\sigma_-\rangle}{dt} = -i\omega_{01}\langle\sigma_-\rangle - if_q\langle\sigma_z\rangle + \frac{i}{\sqrt{N_s}}\sum_k e^{ikr} g_k \langle a_k \sigma_z\rangle,$$

$$\frac{d\langle\sigma_z\rangle}{dt} = 2if_q\left(\langle\sigma_+\rangle - \langle\sigma_-\rangle\right) + \frac{2i}{\sqrt{N_s}}\sum_k \left(e^{-ikr} g_k^* \langle a_k^\dagger \sigma_-\rangle - e^{ikr} g_k \langle a_k \sigma_+\rangle\right).$$

…

III. Классический сигнал и источник состояний «0 + 1»

Приближение Маркова-Борна:

$$\frac{d\langle\sigma_-\rangle}{dt} = \langle\sigma_-\rangle\left(-i\omega_{01} - \gamma\right) - \frac{i\Omega_1}{2}e^{-i\omega_1 t}\langle\sigma_z\rangle + i\sqrt{\gamma\gamma_e}\langle\sigma_z\sigma_-^e\rangle,$$

$$\frac{d\langle\sigma_z\rangle}{dt} = -\Gamma(\langle\sigma_z\rangle + 1) + i\Omega_1\left(\langle\sigma_+\rangle e^{i\omega_1 t} - \langle\sigma_-\rangle e^{-i\omega_1 t}\right) + 2i\sqrt{\gamma\gamma_e}(\langle\sigma_-\sigma_+^e\rangle - \langle\sigma_+\sigma_-^e\rangle).$$

Состояния эмиттера:

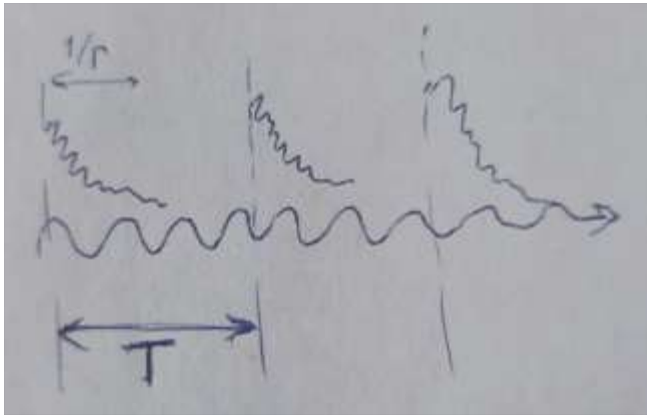
$$\sqrt{1-\nu}\,|\downarrow\rangle + e^{-i\omega_{01}^e Tn}\sqrt{\nu}\,|\uparrow\rangle$$

где ν - вероятность эмиссии фотона

$$\langle\sigma_-^e(Tn)\rangle = \frac{\sin\theta}{2}e^{-i\omega_{01}^e Tn},$$

$$\cos\theta = 2\nu - 1$$

$$\langle\sigma_z^e(Tn)\rangle = \cos\theta,$$



Состояния рассеивателя (период T большой):

$$\langle\sigma_z(Tn)\rangle = -\left(1 + \frac{\gamma}{\Gamma}\frac{\Omega_1^2}{(\Delta\omega)^2 + \gamma^2}\right)^{-1},$$

$$\langle\sigma_-(Tn)\rangle = \frac{1}{2}\frac{\Omega_1 e^{-i\omega_1 Tn}}{\Delta\omega - i\gamma}\left(1 + \frac{\gamma}{\Gamma}\frac{\Omega_1^2}{(\Delta\omega)^2 + \gamma^2}\right)^{-1}.$$

III. Классический сигнал и источник состояний «0 + 1»

Перекрестные корреляторы:

$$\langle \sigma_z \sigma_-^e(Tn)\rangle = \langle \sigma_z(Tn)\rangle\langle \sigma_-^e(Tn)\rangle = -\left(1+\frac{\gamma}{\Gamma}\frac{\Omega_1^2}{(\Delta\omega)^2+\gamma^2}\right)^{-1}\frac{\sin\theta}{2}e^{-i\omega_{01}^e Tn}.$$

$$\langle \sigma_- \sigma_+^e(Tn)\rangle = \langle \sigma_-(Tn)\rangle\langle \sigma_+^e(Tn)\rangle = \frac{1}{2}\frac{\Omega_1}{\Delta\omega - i\gamma}\left(1+\frac{\gamma}{\Gamma}\frac{\Omega_1^2}{(\Delta\omega)^2+\gamma^2}\right)^{-1}\frac{\sin\theta}{2}e^{i(\omega_{01}^e-\omega_1)Tn}.$$

Уравнения движения на корреляторы:

$$\frac{d\langle \sigma_z\sigma_-^e\rangle}{dt} = \langle \sigma_z\sigma_-^e\rangle(-i\omega_{01}^e - \Gamma - \gamma_e),$$

$$\frac{d\langle \sigma_-\sigma_+^e\rangle}{dt} = \langle \sigma_-\sigma_+^e\rangle\big(i(\omega_{01}^e - \omega_{01}) - \gamma - \gamma_e\big).$$

Приближенное решение:

$$\langle \sigma_- \rangle = c_1 e^{i\delta\omega t} + c_{-1} e^{-i\delta\omega t} + c_{-3} e^{-3i\delta\omega t}.$$

Структура спектра:

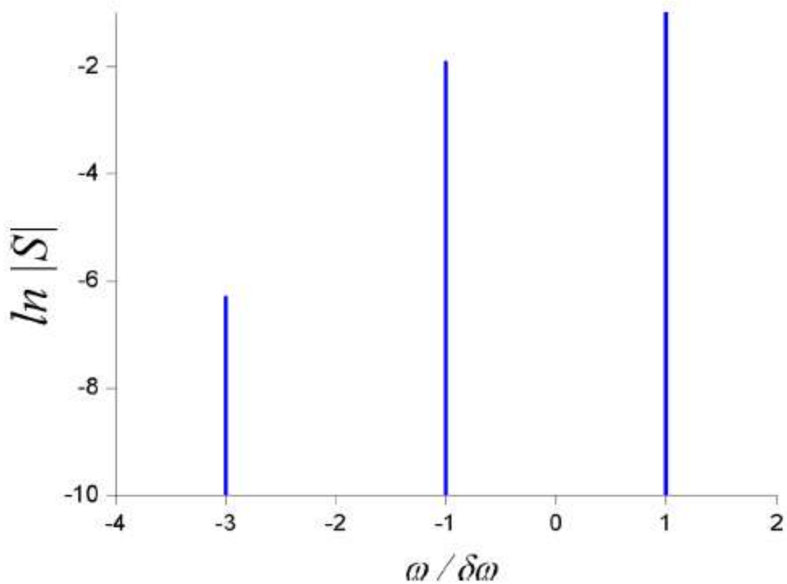


FIG. 4. Spectral components of $\langle \sigma_- \rangle$ in the case of qubit irradiation by a coherent wave together with the superposition of Fock states with 0 and 1 photon (see text).

Единственный многофотонный процесс:

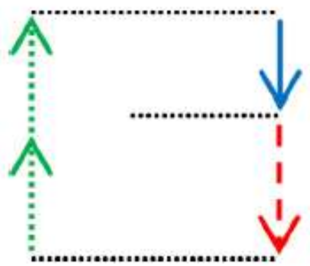


FIG. 5. Schematic image of a multiphoton process resulting in a side peak in the emission spectra at $3\delta\omega$ under the qubit irradiation by the classical signal and quantum superpositions of Fock states with 0 and 1 photons. Green arrows (dotted lines) correspond to photons from the coherent wave with energies ω_1. Blue arrow (solid line) corresponds to the single photon with energy ω_2. Red arrow (dashed line) indicates photon emission with energy $2\omega_1 - \omega_2 = \omega_d + 3\delta\omega$.

IV. Wave-mixing in a cascaded two-qubit system

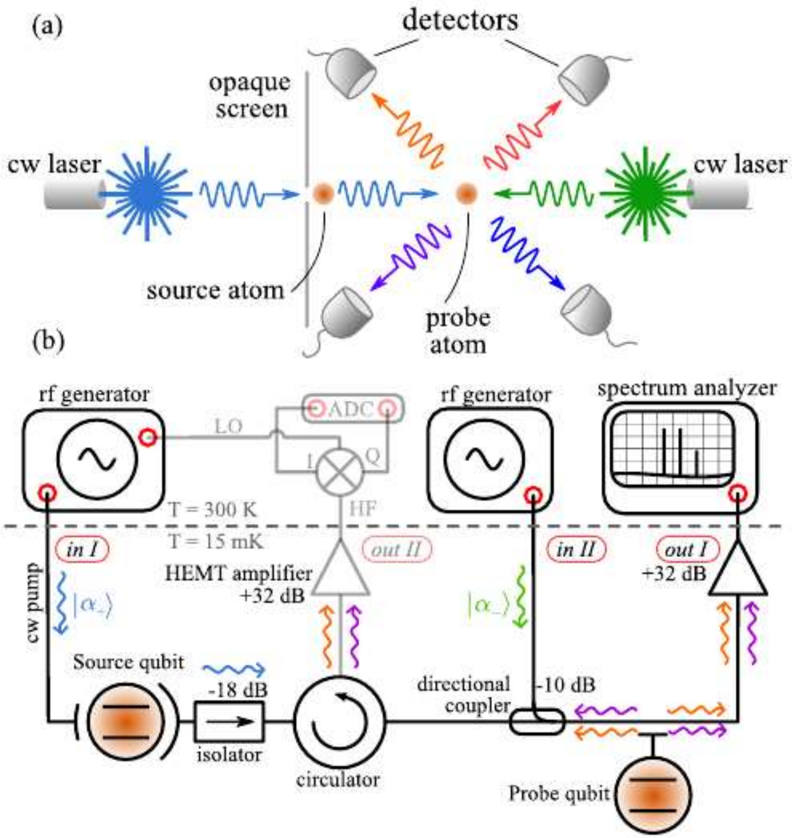


FIG. 1. (a) The experiment's optical concept: the probe atom scatters two coherent fields—non-classical from a source through a small aperture in an opaque screen, and classical from an external generator. The probe's field is detected and analyzed. (b) Simplified sketch of the waveguide-QED microwave setup with two superconducting transmon qubits in a dilution refrigerator. The measurements in this work are done with the use of the channel labeled as "out I".

- Source atom: light antibunched in the time domain

- Second-order correlation function:

$$g^{(2)}(\tau) = \frac{\langle \sigma_+(0)\sigma_+(\tau)\sigma_-(\tau)\sigma_-(0)\rangle}{((1+\langle\sigma_z\rangle)/2)^2}.$$

$$g^{(2)}(\tau) = 1 - e^{-\frac{3}{4}\gamma\tau}\left[\cos(\Omega_g\tau) + \frac{3\gamma}{\Omega_g}\sin(\Omega_g\tau)\right]$$

Thus, $g^{(2)}(0)$ is always equal to 0.

IV. Wave-mixing in a cascaded two-qubit system

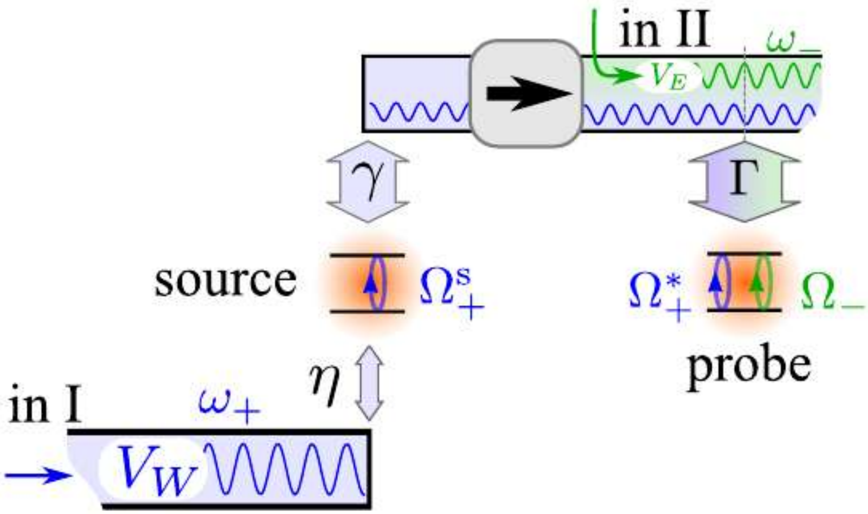


FIG. 5. The schematic image of theoretically described two-atomic cascade. Semi-infinite waveguides are shown by half-cut rectangles, couplings are shown as two-sided arrows.

We need to take into account uni-directional coupling between the
source and the probe via the waveguide

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\mathrm{q}} + \hat{\mathcal{H}}_{\mathrm{dr,s}} + \hat{\mathcal{H}}_{\mathrm{dr,p}}$$

$$\hat{\mathcal{H}}_{\mathrm{q}} = \frac{1}{2}\left(\omega_s \sigma_s^z + \omega_p \sigma_p^z\right)$$

$$\hat{\mathcal{H}}_{\mathrm{dr,s}} = -i\sqrt{\eta}\left(W\hat{\sigma}_s^+ e^{i(\omega_d-\delta\omega)t} + \mathrm{c.c.}\right)$$

$$\hat{\mathcal{H}}_{\mathrm{dr,p}} = -i\sqrt{\frac{\Gamma}{2}}\left(E\hat{\sigma}_p^+ e^{i(\omega_d+\delta\omega)t} + \mathrm{c.c.}\right)$$

[1] C. W. Gardiner and A. S. Parkins, Phys. Rev. A **50**, 1792 (1994).

IV. Wave-mixing in a cascaded two-qubit system

Now we need to include not only the radiative relaxation of each atom, but also the fact that the probe is irradiated by all the field emitted by the source. As shown [39], this task is accomplished by the standard form of the quantum master equation for the two-qubit density matrix ρ:

$$\frac{\partial}{\partial t}\rho = -i\left[\hat{\mathcal{H}}, \rho\right] + \hat{\mathcal{L}}\rho. \tag{12}$$

but with a very specific Lindblad term with a following form:

$$\hat{\mathcal{L}}\rho = \hat{\mathcal{L}}_s\rho + \hat{\mathcal{L}}_p\rho + \hat{\mathcal{L}}_{sp}\rho. \tag{13}$$

The first two terms are straightforward and they describe the radiational relaxation of atoms because of waveguide modes:

$$\hat{\mathcal{L}}_{s,p}\rho = \hat{L}_{s,p}\rho\hat{L}^{\dagger}_{s,p} - \frac{1}{2}\left\{\hat{L}^{\dagger}_{s,p}\hat{L}_{s,p}, \rho\right\},$$
$$\hat{L}_s = \sqrt{\gamma+\eta}\sigma_s^-, \hat{L}_p = \sqrt{\Gamma}\sigma_p^-, \tag{14}$$

whereas the coupling term is non-hermitian and has the form:

$$\hat{\mathcal{L}}_{sp}\rho = \alpha\sqrt{\gamma\Gamma}\left(\left[\sigma_s^-\rho, \sigma_p^+\right] + \left[\sigma_p^-, \rho\sigma_s^+\right]\right). \tag{15}$$

[1] C. W. Gardiner and A. S. Parkins, Phys. Rev. A **50**, 1792 (1994).

IV. Wave-mixing in a cascaded two-qubit system

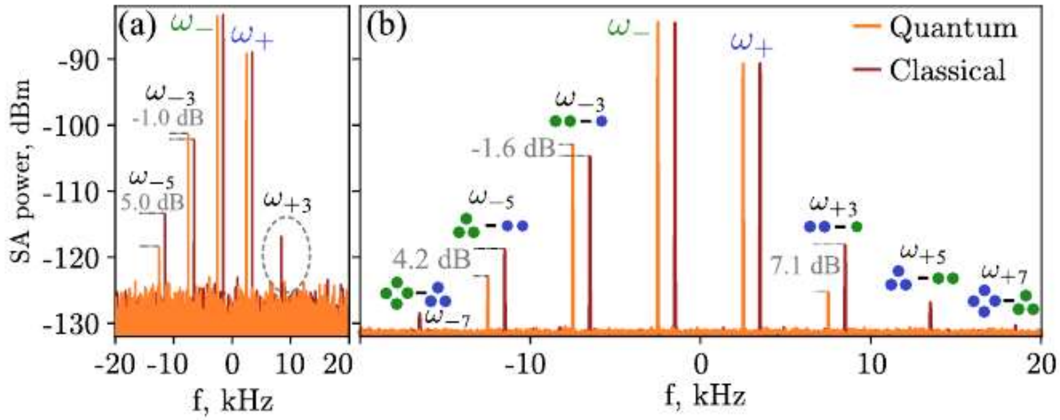


FIG. 6. (a) Single measurement of coherent components for cascaded mixing at point $\nu_+/\gamma = -9.9$ dB and $\nu_-/\gamma = -4.0$ dB (orange trace) and for classical mixing (brown trace) for nearly same driving parameters, selected in a way that emitted components at $\omega_\pm$ are maximally equal. The difference in side peaks at ω_{-3} and ω_{-5} is labeled, and the absence of ω_{+3}-component in cascaded mixing is highlighted with dashed oval. (b) The comparison of classical and quantum (cascaded) traces averaged over several values of driving strengths, around the parameters specified in (a). The classical trace is shift by 1 kHz for clear comparison. The difference of corresponding components is labeled.

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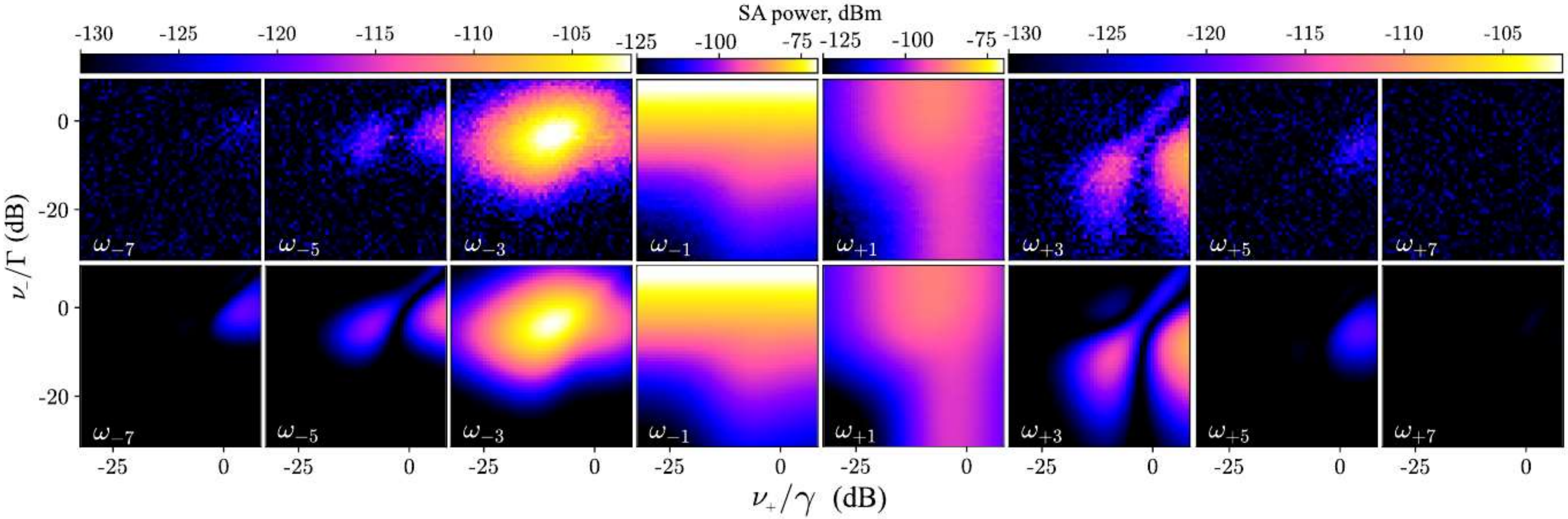


FIG. 7. The side components emitted by the probe in wave mixing processes of stationary atomic emission at ω_+ (emitted by the driven source) and classical tone at ω_-, both applied to the probe. Upper row of panels: power of each side peak measured by spectral analyzer with a resolution bandwidth of 5 Hz as a function of ν_+/γ and ν_-/Γ. Lower row of panels: the power fit obtained through numerical calculations. The fitting parameters are $\Gamma/2\pi = 1.8$ MHz, $\gamma/2\pi = 1.7$ MHz, $\alpha = 0.79$ and some general attenuation and amplification coefficients.

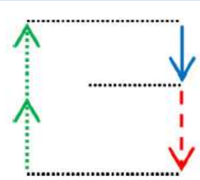
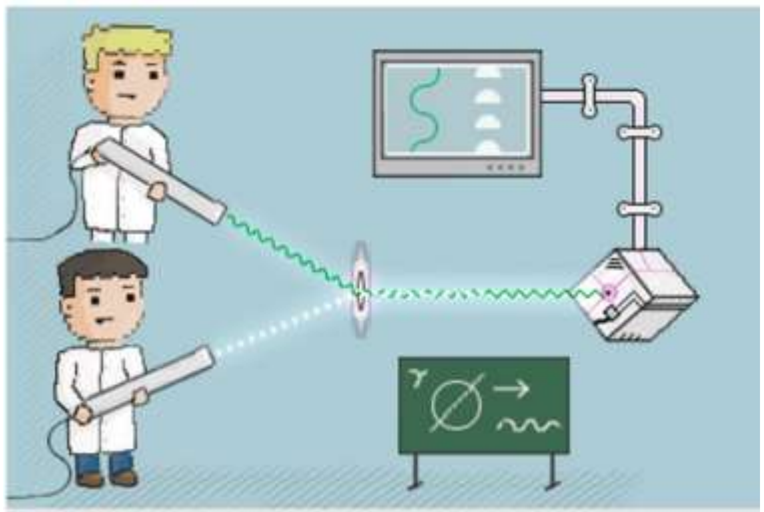


FIG. 5. Schematic image of a multiphoton process resulting in a side peak in the emission spectra at $3\delta\omega$ under the qubit irradiation by the classical signal and quantum superpositions of Fock states with 0 and 1 photons. Green arrows (dotted lines) correspond to photons from the coherent wave with energies ω_1. Blue arrow (solid line) corresponds to the single photon with energy ω_2. Red arrow (dashed line) indicates photon emission with energy $2\omega_1 - \omega_2 = \omega_d + 3\delta\omega$.

Summary

- Wave mixing of classical and nonclassical signals on a single qubit
- The spectrum of the scattered wave differs from the case of mixing two classical signals
- The structure of the spectrum can indicate the photon statistics in the nonclassical mode



Физики смешивают классический свет и полфотона на кубите. Дизайнер Павел Одинев/ Сколтех

W. V. Pogosov, A. Yu. Dmitriev, and O. V. Astafiev, "*Effects of photon statistics in wave mixing on a single qubit*", Phys. Rev. A 104, 023703 (2021); arXiv:2107.10769.

A. Yu. Dmitriev, A. V. Vasenin, S. A. Gunin, S. V. Remizov, A. A. Elistratov, W. V. Pogosov, and O. V. Astafiev, "*Direct experimental observation of sub-poissonian photon statistics by means of multi-photon scattering on a two-level system*", submitted to Phys. Rev. A; arXiv:2409.10975.

$$\langle a^\dagger_{out}(t)a_{out}(t')\rangle = Ne^{i\omega_2(t-t')}\delta(t-t'), \tag{9}$$

$$\langle a_{out}(t)a_{out}(t')\rangle = Me^{-i\omega_2(t+t')}\delta(t-t'), \tag{10}$$

where M is a measure of light squeezing. In general, $|M|^2 \leq N(N+1)$, while $|M|^2 = N(N+1)$ corresponds to the pure squeezed state.