All fractional Shapiro steps in the RSJ model with two Josephson harmonics

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Synchronization between the internal dynamics of the superconducting phase in a Josephson junction (JJ) and an external ac signal is a fundamental physical phenomenon, manifesting as constant-voltage Shapiro steps in the current-voltage characteristic. Mathematically, this phase-locking effect is captured by the Resistively Shunted Junction (RSJ) model, an important example of a nonlinear dynamical system. The standard RSJ model considers an overdamped JJ with a sinusoidal (single-harmonic) current-phase relation (CPR) in the current-driven regime with a monochromatic ac component. While this model predicts only integer Shapiro steps, the inclusion of higher Josephson harmonics is known to generate fractional Shapiro steps. In this paper, we show that only two Josephson harmonics in the CPR are sufficient to produce all possible fractional Shapiro steps within the RSJ framework. Using perturbative methods, we analyze amplitudes of these fractional steps. Furthermore, by introducing a phase shift between the two Josephson harmonics, we reveal an asymmetry between positive and negative fractional steps — a signature of the Josephson diode effect.

Bibliography
[1] Pavel N. Tsarev and Yakov V. Fominov, arXiv:2505.20502.