Physics of surfaces and interfaces

3. Surface, edge and domain-wall superconductivity in nanostructured hybrid samples

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Outline of current lecture

* Introduction
* Bulk superconductivity and surface superconductivity in parallel magnetic field
* Edge superconductivity in perpendicular magnetic field
* Domain-wall superconductivity
* Superconductivity at twin boundaries

Introduction:
Ginzburg-Landau formalism

Ginzburg-Landau functional of free energy (1)

In order to describe local superconducting properties we introduce a complex-valued function – so-called superconducting order parameter $\Psi(\boldsymbol{r})$

$$|\Psi(\boldsymbol{r})|^2 \equiv \frac{n_s(\boldsymbol{r})}{2},$$

where $n_s(\boldsymbol{r})/2$ is the local density of superconducting electrons (i. e. Cooper pairs).

According to the Landau's theory of second-order phase transitions, the density of free energy of metal in superconducting state at given temperature T can be expanded in power series with respect to $|\Psi|^2$ and $|\nabla\Psi|^2$

$$f_s = f_{n,0} + \alpha\,|\Psi|^2 + \frac{\beta}{2}\,|\Psi|^4 + \frac{1}{4m^*}\left|\Big(-i\hbar\nabla - \frac{2e}{c}\,\boldsymbol{A}\Big)\,\Psi\right|^2 + \frac{\boldsymbol{B}^2}{8\pi}$$

where f_{n0} is the density of free energy in normal state in zero magnetic field, $\boldsymbol{A}(\boldsymbol{r})$ is vector potential, $\boldsymbol{B}(\boldsymbol{r}) = rot\ \boldsymbol{A}(\boldsymbol{r})$ is local magnetic field (more precisely, magnetic flux density), $2e$ and $2m^*$ are charge and effective mass of the Cooper pair ($e = -|e|$).

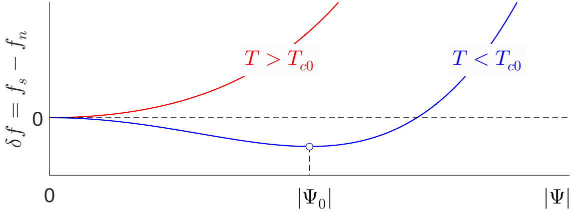
The coefficient α in the vicinity of the critical temperature of the phase transition T_{c0} has the following temperature dependence: $\alpha = const \cdot (T - T_{c0})$; while the coefficient β can be considered as temperature-independent.

Ginzburg-Landau functional of free energy (2)

The difference of the free-energy density for an uniform superconducting state ($\Psi = \Psi_0$) and free-energy density for normal state ($\Psi = 0$) in absence of magnetic field and current is given by

$$\delta f \equiv f_s - f_n = \alpha \left|\Psi_0\right|^2 + \frac{\beta}{2} \left|\Psi_0\right|^4.$$

If $T > T_{c0}$ and $\alpha > 0$, then the minimum of δf corresponds to the normal state: $\Psi_0 = 0$ and the appearance of superconductivity is energetically unfavourable. If $T < T_{c0}$ and $\alpha < 0$, then the minimum of δf corresponds to nonzero value of the order parameter: $|\Psi_0| = \sqrt{-\alpha/\beta} \propto \sqrt{1 - T/T_{c0}}$.



For convenience we can introduce the dimensionless order parameter $\psi(\boldsymbol{r}) \equiv \Psi(\boldsymbol{r})/|\Psi_0|$ normalized at its equilibrium value at given temperature.

Ginzburg-Landau functional of free energy (3)

In presence of external magnetic field $\boldsymbol{H}_{ext}(\boldsymbol{r})$ is it logical to consider the density of thermodynamic potential

$$g_s = f_{n,0} + \alpha\,\Psi_0^2\,|\psi|^2 + \frac{\beta}{2}\,\Psi_0^4\,|\psi|^4 + \frac{\Psi_0^2}{4m^*}\left|\left(-i\hbar\nabla - \frac{2e}{c}\,\boldsymbol{A}\right)\psi\right|^2 + \frac{rot^2\,\boldsymbol{A}}{8\pi} - \frac{rot\,\boldsymbol{A}\cdot\boldsymbol{H}_{ext}}{4\pi}$$

and full thermodynamic potential (Gibbs free energy)

$$G \equiv \iiint g_s\,dV = \iiint f_{n,0}\,dV +$$

$$\iiint \frac{\Phi_0^2}{32\pi^3\lambda^2\xi^2}\left\{-|\psi|^2 + \frac{|\psi|^4}{2} + \xi^2\left|\left(-i\nabla + \frac{2\pi}{\Phi_0}\,\boldsymbol{A}\right)\psi\right|^2 + \frac{rot^2\,\boldsymbol{A}}{8\pi} - \frac{rot\,\boldsymbol{A}\cdot\boldsymbol{H}_{ext}(\boldsymbol{r})}{4\pi}\right\} dV.$$

Here $\Phi_0 = \pi\hbar c/|e| = 2{\cdot}10^{-7}$ Oe$\cdot$cm^2 is the magnetic flux quantum, $\lambda = \sqrt{m^* c^2\beta/8\pi e^2|\alpha|}$ is the London penetration depth, $\xi = \sqrt{\hbar^2/4m^*|\alpha|}$ is the coherence length. Taking into account $\alpha = const\cdot(T - T_{c0})$, we get universal temperature dependences

$$\lambda(T) = \frac{\lambda_0}{\sqrt{1 - T/T_{c0}}} \qquad \text{and} \qquad \xi(T) = \frac{\xi_0}{\sqrt{1 - T/T_{c0}}},$$

where λ_0 and ξ_0 are the London penetration depth and the coherence length at $T = 0$.

Ginzburg-Landau (GL) equations

By minimizing G with respect to ψ^* and $\boldsymbol{A}$, we derive the system of equations (so-called Gunzburg-Landau equations)

$$\xi^2 \left(-i\nabla + \frac{2\pi}{\Phi_0}\boldsymbol{A}\right)^2 \psi - \psi + |\psi|^2 \psi = 0,$$

$$rot\ rot\ \boldsymbol{A} = \frac{4\pi}{c}\boldsymbol{j}_s + \frac{4\pi}{c}\boldsymbol{j}_{ext}$$

where the densities of superconducting current and external current are equal to

$$\boldsymbol{j}_s = \frac{c}{4\pi}\frac{|\psi|^2}{\lambda^2}\left(-\frac{\Phi_0}{2\pi}\nabla\theta - \boldsymbol{A}\right) \quad \text{и} \quad \boldsymbol{j}_{ext} = \frac{4\pi}{c}\, rot\, \boldsymbol{H}_{ext},$$

$\theta(\boldsymbol{r}) = \arg\psi(\boldsymbol{r})$ is the phase of the order parameter.

For interfaces superconductor-vacuum or superconductor-insulator the boundary conditions for the order parameter take the form

$$\left(-i\nabla + \frac{2\pi}{\Phi_0}\boldsymbol{A}(\boldsymbol{r})\right)_{\mathbf{n}}\psi(\boldsymbol{r}) = 0 \quad \text{or} \quad i\,\frac{\partial}{\partial n}|\psi(\boldsymbol{r})| + \left(-\frac{\partial}{\partial n}\theta(\boldsymbol{r}) - \frac{2\pi}{\Phi_0}A_n(\boldsymbol{r})\right)|\psi(\boldsymbol{r})| = 0$$

where $\boldsymbol{n}$ is a normal vector (perpendicular to the surface)

Linearized Ginzburg-Landau equation (1)

At initial stage of formation of superconductivity one can assume that the density of superconducting condensate is very small: $|\psi|^2 \ll 1$. It makes possible to neglect non-linear terms in the GL equations and consider a decoupled linearized GL equation in given magnetic field

$$\left(-i\nabla + \frac{2\pi}{\Phi_0}\,\boldsymbol{A}\right)^2 \psi = \frac{1}{\xi^2}\,\psi, \quad \text{where} \quad \boldsymbol{A}(\boldsymbol{r}) = \frac{1}{c}\iiint \frac{\boldsymbol{j}_{ext}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,d\boldsymbol{r}'.$$

The linearized GL equation coincides with the Schrödinger equation for free spin-less particle in the magnetic field

$$\frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e}{c}\,\boldsymbol{A}\right)^2 \psi = E\,\psi,$$

where E is total energy.

Intermediate conclusion:
This formal analogy between the linearized GL equation and the Schrödinger equation helps us to map problems considered in university-level quantum mechanics into physics of superconductors.

Linearized Ginzburg-Landau equation (2)

The equation

$$\left(-i\nabla + \frac{2\pi}{\Phi_0}\,\boldsymbol{A}\right)^2 \psi = \frac{1}{\xi^2}\,\psi$$

is equivalent to the Sturm-Liouville problem for determination of eigenfunctions $\psi_n(\boldsymbol{r})$ and eigenvalues $E_n = (1/\xi^2)_n$ of the of the differential operator $\hat{H} = \big(-i\nabla + 2\pi\,\boldsymbol{A}/\Phi_0\big)^2$.

Since

$$\xi(T) = \frac{\xi_0}{\sqrt{1 - T/T_{c0}}} \quad \Longrightarrow \quad \frac{1}{\xi^2} = \frac{1}{\xi_0^2}\cdot\left(1 - \frac{T}{T_{c0}}\right),$$

one can define a critical temperature for each type of the eigenfunctions

$$\left(\frac{1}{\xi^2}\right)_n = \frac{1}{\xi_0^2}\cdot\left(1 - \frac{T_{c,n}}{T_{c0}}\right) \quad \text{or} \quad T_{c,n} \equiv T_{c0}\cdot\Big\{1 - \xi_0^2\cdot\left(1/\xi^2\right)_n\Big\}.$$

Here T_{c0} is the critical temperature of bulk sample at zero magnetic field ($B = 0$).

The critical temperature of the superconducting transition can be defined as a maximum among all $T_{c,n}$ values

$$T_c \equiv \max T_{c,n} = T_{c0}\cdot\Big\{1 - \xi_0^2\cdot\min\,\left(1/\xi^2\right)_n\Big\}.$$

In the order words, the critical temperature is defined by the lowest eigenvalue of the Schrödinger-like equation.

Quantum-size effect for superconducting condensate

Particle-in-the-box problem in quantum mechanics

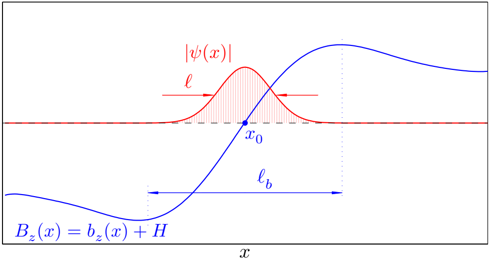
$$E_n \sim \frac{\hbar^2}{2m} \cdot \frac{n^2}{\ell^2} \quad \Longrightarrow \quad min\left(\frac{2mE_n}{\hbar^2}\right) \sim \frac{1}{\ell^2},$$

where ℓ is the localization radius of single-electron wave function.

Comparison of quantum mechanics and theory of localized superconductivity

$$min\left(\frac{2mE_n}{\hbar^2}\right) \equiv \frac{1}{\xi_0^2}\left(1 - \frac{T_c}{T_{c0}}\right) \quad \Longrightarrow \quad 1 - \frac{T_c}{T_{c0}} \simeq \frac{\xi_0^2}{\ell^2}.$$

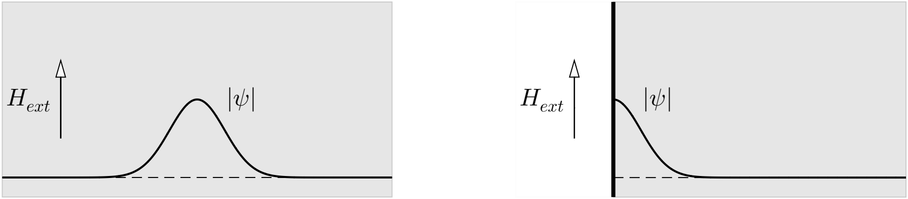
where ℓ is the localization radius of the order parameter wave function.



The smaller the ℓ value, the lower the critical temperature T_c is and vice versa.

Part I

Bulk and surface superconductivity in parallel magnetic field



Nucleation of superconductivity in bulk samples (1)

Let us choose the $z-$axis along the external magnetic field ($\boldsymbol{H}_{ext} = H\,\boldsymbol{e}_z$), then the vector potential has the form $\boldsymbol{A} = Hx\,\boldsymbol{e}_y$. In this case the effective Hamiltonian

$$\hat{H} = \left(-i\nabla + \frac{2\pi}{\Phi_0}\,\boldsymbol{A}\right)^2 = \left(-i\frac{\partial}{\partial x}\,\boldsymbol{e}_x - i\frac{\partial}{\partial y}\,\boldsymbol{e}_y - i\frac{\partial}{\partial z}\,\boldsymbol{e}_z + \frac{2\pi}{\Phi_0}\,Hx\,\boldsymbol{e}_y\right)^2$$

is invariant with respect to parallel displacements along $y-$ and $z-$axes; therefore $y-$ and $z-$components of the momentum should be conserved.

This allows to seek the solution of the linearized GL equation $\hat{H}\,\psi = \xi^{-2}\,\psi$ in the form

$$\psi(\boldsymbol{r}) = f(x)\cdot e^{-iky}\cdot e^{-iqz}.$$

We start with the analysis of the nucleation of superconductivity in bulk samples far from the boundaries. We restrict ourselves by seeking the localized solutions with maxima in the inner part of the sample and vanishing at $x\to\pm\infty$.

The boundary value problem can be formulated as follows

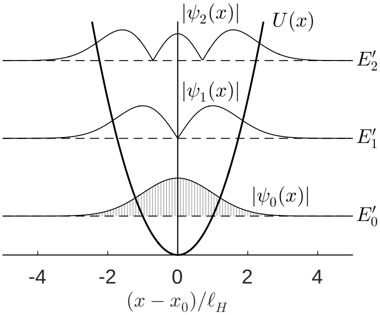
$$-f''(x) + \left(\frac{2\pi}{\Phi_0}\,Hx - k\right)^2 f(x) + q^2\,f(x) = \frac{1}{\xi^2}\,f(x) \quad \text{and} \quad f(x)\Big|_{x\to\pm\infty} = 0.$$

Nucleation of superconductivity in bulk samples (2)

This problem

$$-f''(x)+\left(\frac{2\pi}{\Phi_0}Hx-k\right)^2 f(x)+q^2 f(x)=\frac{1}{\xi^2}f(x) \quad \text{and} \quad f(x)\Big|_{x\to\pm\infty}=0$$

is equivalent to the classical problem for the determination of the eigenfunctions and eigenvalues for a quantum particle in parabolic potential well $U(x)=(2\pi\,H\,x/\Phi_0-k)^2$, shifted from the origin at $x_0=\Phi_0 k/(2\pi H)$.



Since the spectrum of quantum-well states do not depend on the position of the well, we conclude that the eigenenergies of our problem ξ^{-2} do not depend on k and x_0.

Nucleation of superconductivity in bulk samples (3)

We would like to use well-known solution for the quantum particle in the parabolic potential well in order to find the conditions for nucleation of bulk superconductivity. The Schroedinger equation for single-electron wave function has the form

$$-\frac{d^2}{dx^2}\,\psi_n(x)+\frac{2m}{\hbar^2}\cdot\frac{m\omega^2x^2}{2}\,\psi_n(x)=\frac{2mE_n}{\hbar^2}\,\psi_n(x)\quad\text{and}\quad E_n=\hbar\omega\,\left(n+\frac{1}{2}\right).$$

The second term in the l.h.s. of the linearized GL equation (page 13) coincides to that in the Schroedinger equation after the replacement $\omega=(\hbar/m)\,(2\pi|H|/\Phi_0)$.

Under this condition the eigenvalues of the linearized GL equation

$$\frac{1}{\xi_0^2}\cdot\left(1-\frac{T_{c,n}}{T_{c0}}\right)-q^2$$

will be equal to the eigenvalues of the Schroedinger equation

$$\frac{2mE_n}{\hbar^2}=\frac{2\pi|H|}{\Phi_0}\,(2n+1)\,.$$

As a result, we come to the relationship between the number n of the quantum-well state and the corresponding critical temperature $T_{c,n}$

$$1-\frac{T_{c,n}}{T_{c0}}=\frac{2\pi\xi_0^2}{\Phi_0}\,|H|\,(2n+1)+\xi_0^2\,q^2.$$

Nucleation of superconductivity in bulk samples (4)

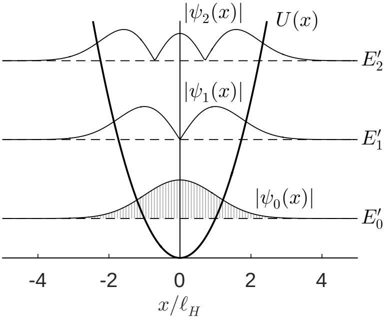
The relationship

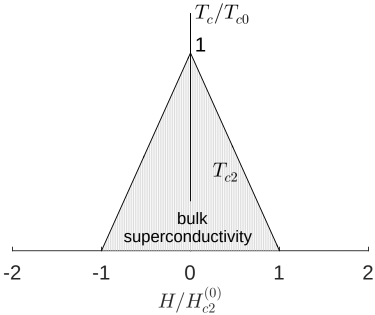
$$1 - \frac{T_{c,n}}{T_{c0}} = \frac{2\pi\xi_0^2}{\Phi_0}\,|H|\,(2n+1) + \xi_0^2\, q^2 .$$

can be used to the determination of the phase transition line putting $n = 0$ and $q = 0$

$$H_{c2} = H_{c2}^{(0)} \cdot \left(1 - \frac{T}{T_{c0}}\right) \quad \text{или} \quad 1 - \frac{T_{c2}}{T_{c0}} = \frac{|H|}{H_{c2}^{(0)}} .$$

where $H_{c2}^{(0)} = \Phi_0/(2\pi\xi_0^2)$ is the upper critical field to zero temperature.





Nucleation of surface superconductivity (1)

We proceed with the estimate of the critical parameters corresponding to the formation of localized superconductivity near flat surface of bulk sample positioned at $x = 0$.

Let us choose the $z-$axis along the external magnetic field ($\boldsymbol{H}_{ext} = H\,\boldsymbol{e}_z$), then the vector potential can be written as $\boldsymbol{A} = Hx\,\boldsymbol{e}_y$. The symmetry of the problem allows to find a solution in the form $\psi(\boldsymbol{r}) = f(x)\cdot e^{-iky}\cdot e^{-iqz}$ (see page 12). Thus, one should determine the minimal eigenvalue of the boundary value problem

$$-f''(x) + \left(\frac{2\pi}{\Phi_0}\,Hx - k\right)^2 f(x) + q^2\,f(x) = \frac{1}{\xi^2}\,f(x)$$

$$\text{provided that}\quad \left.\frac{df}{dx}\right|_{x=0} = 0 \quad\text{and}\quad f(x)\Big|_{x\to\infty} = 0.$$

Unfortunately, this problem has no analytical solution, therefore one should apply various methods of approximate calculations like the variational principle (see lecture 1)

$$E_{app} = \frac{\int \psi^*_{app}(\boldsymbol{r})\,\hat{H}\psi_{app}(\boldsymbol{r})\,dV}{\int \psi^*_{app}(\boldsymbol{r})\,\psi_{app}(\boldsymbol{r})\,dV}.$$

Nucleation of surface superconductivity (2)

We assume that

$$f(x) = e^{-x^2/2a^2} \quad \text{and} \quad \psi_{app}(\boldsymbol{r}) = e^{-x^2/2a^2} \cdot e^{-iky} \cdot e^{-iqz},$$

where a is a width of the superconducting gaussian-like function. Thus, we come to the problem of minimization of the function

$$E_{app} = \frac{\int \psi^*_{app}(\boldsymbol{r})\, \hat{H}\psi_{app}(\boldsymbol{r})\, dV}{\int \psi^*_{app}(\boldsymbol{r})\, \psi_{app}(\boldsymbol{r})\, dV} = \frac{1}{2a^2} + \frac{1}{\ell_H^4}\,\frac{a^2}{2} - \frac{ka}{\ell_H^2}\,\frac{2}{\sqrt{\pi}} + k^2 + q^2,$$

which depends on three independent parameters k, q and a. Here for sake of convenience we introduce so-called magnetic length $\ell_H \equiv \sqrt{\Phi_0/(2\pi|H|)}$.

The minimum of E_{app} for the considered class of function is equal to

$$E_{min} \equiv E_{app}(k_{min}, q_{min}, a_{min}) = \frac{\sqrt{(\pi-2)/\pi}}{\ell_H^2} \simeq \frac{0.603}{\ell_H^2},$$

where

$$k_{min} = \frac{1}{\ell_H}\sqrt[4]{\frac{1}{\pi(\pi-2)}}, \quad q_{min} = 0 \quad \text{and} \quad a_{min} = \ell_H\sqrt[4]{\frac{\pi}{(\pi-2)}}.$$

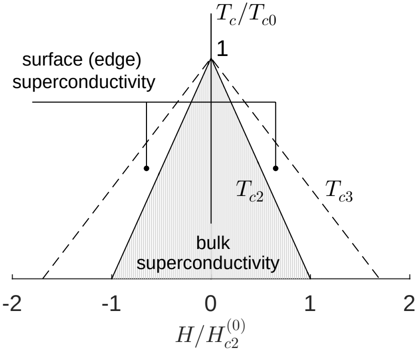
Nucleation of surface superconductivity (3)

Analytical solution vs. Numerical solution

$$E_{min} \simeq \frac{0.603}{\ell_H^2} \qquad vs. \qquad E_{min} \simeq \frac{0.590}{\ell_H^2}.$$

As a result, the critical field of appearance of surface superconductivity is equal

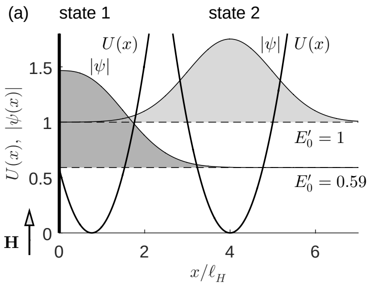
$$E_{min} = \frac{1}{\xi_0^2}\left(1 - \frac{T_c}{T_{c0}}\right) \simeq \frac{0.590}{\ell_H^2} \quad \Longrightarrow \quad H_{c3} = 1.695 \cdot H_{c2}^{(0)} \cdot \left(1 - \frac{T}{T_{c0}}\right).$$

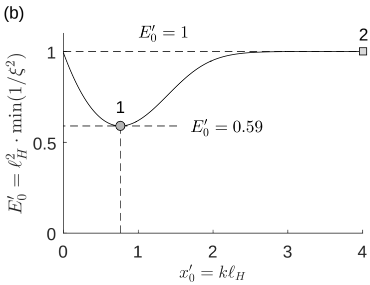


At $H_{c2} < |H| < H_{c2}$ superconductivity exists in the form of surface superconductivity.

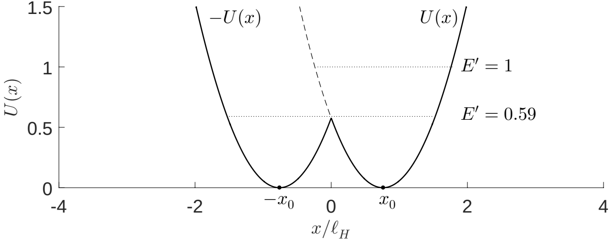
Nucleation of surface superconductivity (4)

Results of numerical simulations



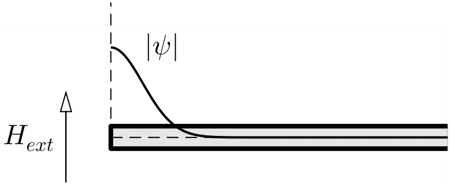


Effective double-well potential



Part II

Edge superconductivity in perpendicular magnetic field



Nucleation of edge superconductivity (1)

Let us discuss the peculiarities of nucleation of superconductivity in thin films in *perpendicular* magnetic field. Assume that (i) the flat edge of the film is positioned at $x = 0$; (ii) the magnetic field and vector potential can be written in the form $\boldsymbol{H}_{ext} = H\,\mathrm{e}_z$ and $\boldsymbol{A} = Hx\,\mathrm{e}_y$.

The symmetry of the problem makes possible to find the solution of the linearized GL equation in the following form

$$\psi(\boldsymbol{r}) = f(x) \cdot e^{-iky} \cdot \Big(A\,\cos qz + B\,\sin qz\Big).$$

Obviously, the function $f(x)$ is the solution of the boundary value problem

$$-f''(x) + \left(\frac{2\pi}{\Phi_0}\,Hx - k\right)^2 f(x) + q^2 f(x) = \frac{1}{\xi^2}\,f(x),$$

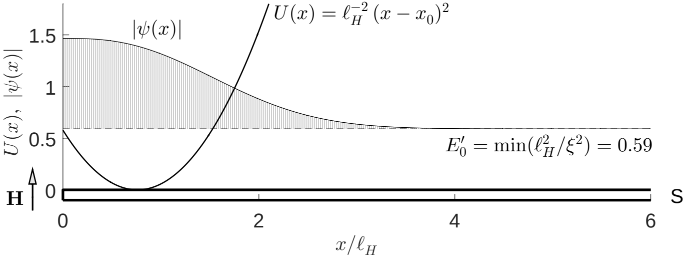
$$\text{provided that}\quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = 0, \quad \psi(x,y,z)\Big|_{x\to\infty} = 0 \quad \text{and} \quad \left.\frac{\partial\psi}{\partial z}\right|_{z=\pm D/2} = 0,$$

where D is the thickness of the superconducting film.

Easy to see that uniform across the film superconducting state ($q = 0$) meets all the boundary conditions and corresponding to the minimum of the eigenenergy $1/\xi^2$.

Nucleation of edge superconductivity (2)

An appearance of uniform across the film superconducting state in thin film with flat edge in the *perpendicular* magnetic field is formally equivalent to the problem of the appearance of surface superconductivity in bulk sample near flat surface in the *parallel* magnetic field.

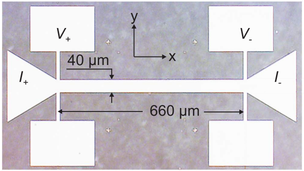


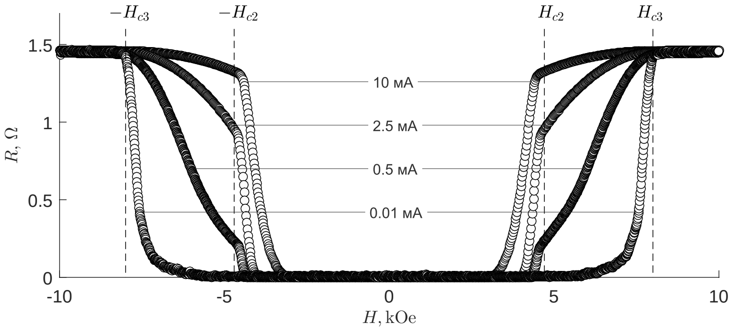
This formal analogy allows us to conclude that the critical parameters for edge superconductivity and for surface superconductivity are the same

$$H_{c3} = 1.695 \cdot H_{c2}^{(0)} \cdot \left(1 - \frac{T}{T_{c0}}\right) \quad \text{или} \quad 1 - \frac{T_{c3}}{T_{c0}} = 0.590 \cdot \frac{|H|}{H_{c2}^{(0)}}.$$

Experimental evidences for edge superconductivity from transport measurements

Dependences of dc-resistance of 40-μm wide superconducting Nb bridge on external magnetic field H, oriented to the perpendicular to the sample plane, and transport current I

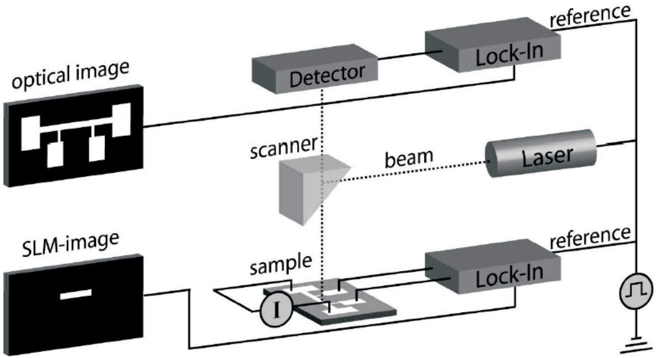


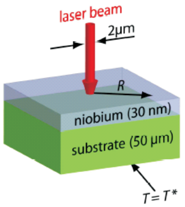


At $I \to 0$ the resistive transition indeed shifts to higher magnetic field exceeding the upper critical field, corresponding to the desctruction/appearance of bulk superconductivity.

Experimental evidences for edge superconductivity from low-temperature scanning laser microscopy (1)

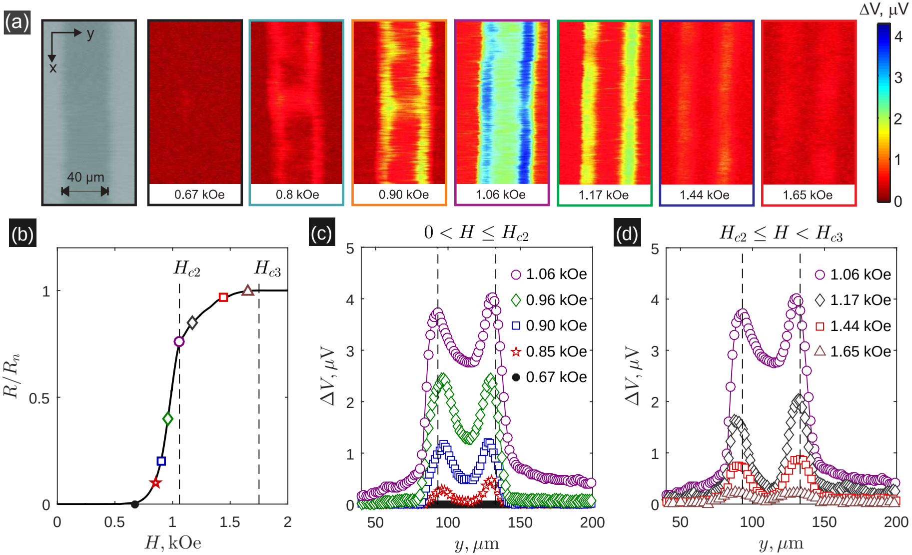
Principle of low-temperature scanning laser microscopy





This figure was taken from J. Fritzsche, Superconductivity controlled by inhomogeneous fields generated by magnetic domains // PhD thesis, Katholieke Universiteit Leuven (2008).

Experimental evidences for edge superconductivity from low-temperature scanning laser microscopy (2)

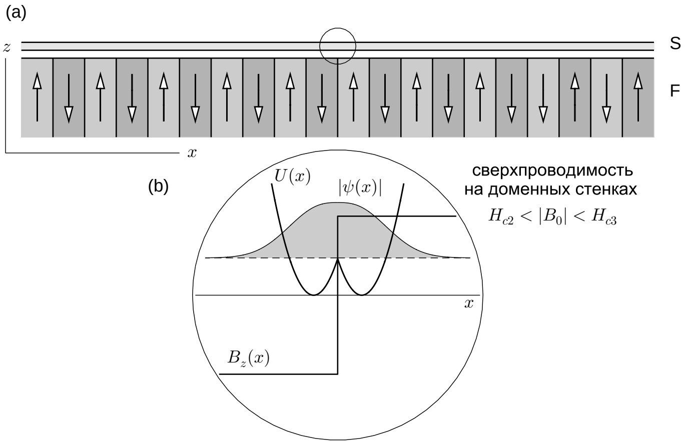


Part III

Domain-wall superconductivity

Domain-wall superconductivity (1)

We consider the nucleation of superconductivity in thin superconducting film infinite in the lateral direction in a spatially modulated magnetic field $B_z(x, y)$, oriented perpendicular to the film. This case is typical for flux-coupled hybrid structures superconductor-ferromagnet with out-of-plane magnetization of the ferromagnetic film.



Domain-wall superconductivity (2)

We assume that a ferromagnetic film has a 1D domain structure. Provided that the thickness of the ferromagnetic film D_f is much larger than the equilibrium width of the domain structure L, then the magnetic field above the ferromagnetic domains is equal

$$B_z(x) = +2\pi M_0 \quad \text{above positively magnetized domains,}$$

$$B_z(x) = -2\pi M_0 \quad \text{above negatively magnetized domains,}$$

where M_0 is the saturated magnetization.

For simplicity we restrict ourselves by analysis of the formation of superconductivity near an isolated domain wall, therefore the local magnetic field and the vector potential in the absence of the external magnetic field can be written as follows

$$\boldsymbol{B} = 2\pi M_0 \, sgn(x) \, \boldsymbol{e}_z \quad \text{and} \quad \boldsymbol{A} = B_0 |x| \, \boldsymbol{e}_y.$$

We already demonstrated that the superconducting state uniform over the film thickness should correspond to the maximal critical temperature. It allows us to seek the solution of the linearized GL equation in the form

$$\psi(\boldsymbol{r}) = f(x) \, e^{-iky}.$$

Domain-wall superconductivity (3)

The function $f(x)$ meets the equation

$$-f''(x) + \left(\frac{2\pi}{\Phi_0}\, 2\pi M_0\, |x| - k\right)^2 f(x) = \frac{1}{\xi^2}\, f(x), \quad \text{provided that} \quad f(x)\Big|_{x\to\pm\infty} = 0.$$

It is easy to see that this boundary problem is invariant with respect to spatial inversion ($x \to -x$), therefore $f(x)$ should be an even function of the $x-$ccordinate. Since the wave function of the ground state in 1D geometry has no zeros and cusps, the first derivative df/dx has to be vanished at the domain wall. All these reasons makes possible to reformulate this boundary problem considering only positive x

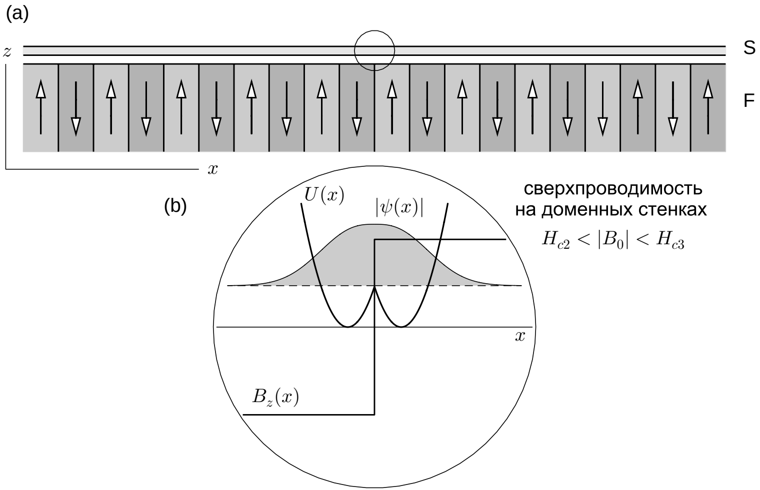
$$-f''(x) + \left(\frac{2\pi}{\Phi_0}\, 2\pi M_0\, x - k\right)^2 f(x) = \frac{1}{\xi^2}\, f(x),$$

$$\text{provided that} \quad \frac{df}{dx}\Big|_{x=0} = 0 \quad \text{and} \quad f(x)\Big|_{x\to\infty} = 0.$$

This problem will be identical to that for the edge superconductivity, if we replace the external magnetic field H by the absolute value of the stray field $B_0 = 2\pi M_0$ produced by magnetic domains, and put $q = 0$.

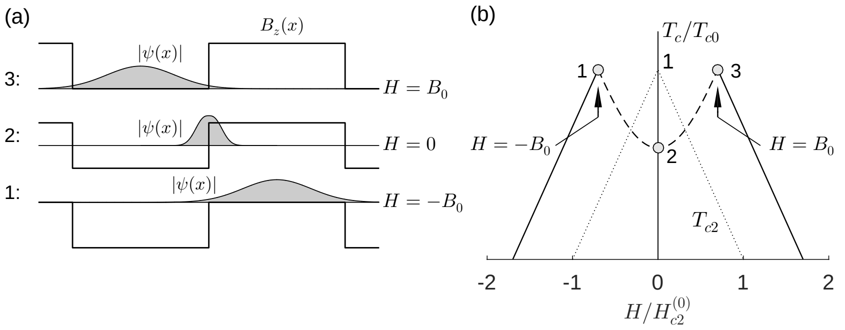
Domain-wall superconductivity (4)

Domain-wall superconductivity at zero external magnetic field



Domain-wall superconductivity (5)

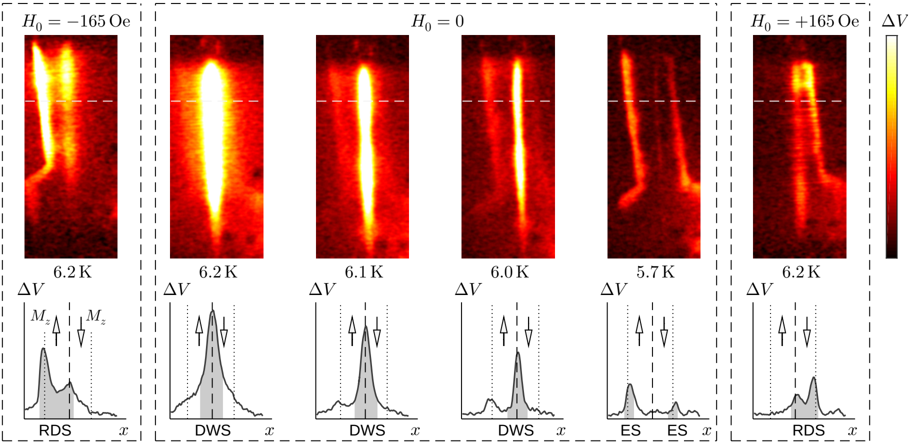
For nonzero external magnetic field H the equivalence between the problems describing the nucleation of edge and domain-wall superconductivity breaks. Nevertheless that if $|H| \leq B_0$ the nucleation of superconductivity near the domain wall is energetically favorable.



The approximate expression for the dependence $T_c^{DWS}(H)$ in the nonuniform magnetic field $B_z(x) = B_0 \, sgn \, x + H$ at $|H| < B_0$

$$1 - \frac{T_c^{DWS}}{T_{c0}} \simeq \frac{B_0}{H_{c2}^{(0)}} \cdot \left\{ 0.59 - 0.70 \left(\frac{H}{B_0} \right)^2 + 0.09 \left(\frac{H}{B_0} \right)^4 \right\}.$$

Experimental evidences for domain-wall superconductivity from low-temperature scanning laser microscopy

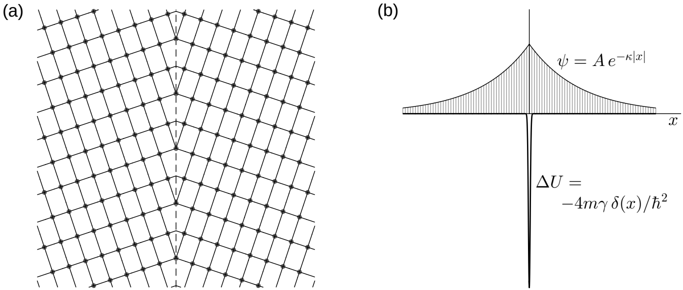


Part IV

Superconductivity at twin boundaries

Superconductivity at twin boundaries (1)

Presence of extended defects of crystalline structure (like twin boundaries) inside bulk superconductors can stimulate the appearance of localized superconductivity at higher magnetic field and temperature as compared with that for uniform bulk samples far from the edges.



The presence of flat single twin boundary at $x = 0$ can be taken into account by extra term in the free energy functional

$$\Delta G = -\iiint \gamma\, |\Psi(\boldsymbol{r})|^2\, \delta(x)\, dV,$$

where $\gamma > 0$ is a phenomenological parameter.

Superconductivity at twin boundaries (2)

After minimization of the functional $G_s + \Delta G$ with respect to Ψ^* one can get a modified boundary value problem in the parallel magnetic field

$$-f''(x) + \left(\frac{2\pi}{\Phi_0} Hx - k\right)^2 f(x) - \frac{4m\gamma}{\hbar^2}\,\delta(x)\,f(x) = \frac{1}{\xi^2}\,f(x),$$

$$\text{provided that} \quad f(x)\Big|_{x\to\pm\infty} = 0.$$

In zero magnetic field ($H = 0$) this problem is equivalent to the problem of the calculation of the Green functions of the following equation

$$f''(x) - k^2 f(x) + \frac{1}{\xi^2} f(x) = -\frac{4m\gamma}{\hbar^2}\,\delta(x)\,f(x).$$

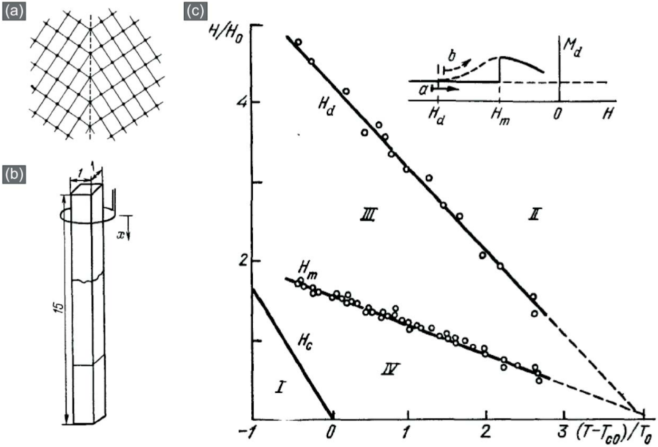
The solution is $f(x) = A\,e^{-\varkappa\,|x|/\hbar^2}$ (homework), where $\varkappa \equiv \sqrt{k^2 - 1/\xi^2}$. After substituting of this solution into the equation, we get

$$\varkappa = \frac{2m\gamma}{\hbar^2} \quad \Longrightarrow \quad \frac{1}{\xi_0^2}\left(1 - \frac{T_c}{T_{c0}}\right) = k^2 - \left(\frac{2m\gamma}{\hbar^2}\right)^2.$$

Superconductivity at twin boundaries (3)

The critical temperature of non-uniform state localized at twin boundary exceed the bulk critical temperature

$$\frac{T_c}{T_{c0}} = 1 + \xi_0^2 \left(\frac{2m\gamma}{\hbar^2}\right)^2 > 1.$$



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