Superconductivity. Phenomenological theory.

Lection 1- introduction

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Textbooks

- A.A.Abrikosov, Fundamentals of the Theory of Metals

- V.V.Schmidt, The Physics of Superconductors: Introduction to Fundamentals and Applications

- M.Tinkham, Introduction to superconductivity

- P. de Gennes, Superconductivity of metals and alloys

- D.Saint-James, G.Sarma, E.J.Thomas, Type II Superconductivity

- V. Mineev, K.Samokhin, Introduction to unconventional superconductivity.
- Ketterson, Song, Superconductivity
- Schrieffer, Theory of superconductivity.
- A.Varlamov, A.Larkin, Theory of Fluctuations in Superconductors

outline

- **Resistivity of metals. Drude model**

- **112 years of superconductivity.**

  Kamerlingh – Onnes (1911)

- **Basic properties of superconductors.**

  current without resistance.

  Magnetic field expulsion. Meissner effect.

- **Thermodynamics of superconductors.**

- **A few words about history and applications.**

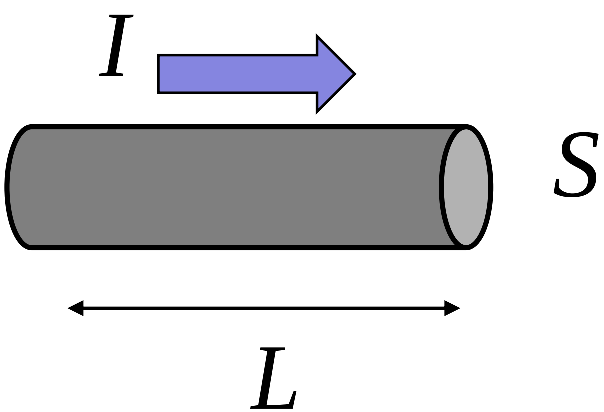
  Magnets, wires, levitation, cryoelectronics …

- **Critical temperature of superconducting transition:**

  higher and higher?

Resistance

$$V = I \times R$$



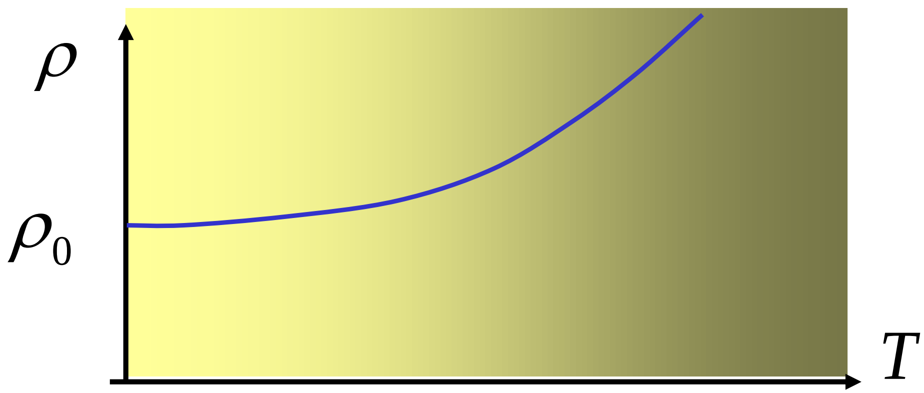
$$R = \rho \frac{L}{S}$$

Conductivity

$$\sigma = \frac{1}{\rho}$$

$$\vec{j} = \sigma \vec{E}$$

Resistance vs temperature



Drude model of conductivity.

$$\vec{V} = \vec{V}_0 + \frac{e\vec{E}t}{m}$$

$$\left\langle \vec{V} \right\rangle = \left\langle \vec{V}_0 \right\rangle + \left\langle \frac{e\vec{E}t}{m} \right\rangle$$

$$\left\langle \vec{V}_0 \right\rangle = 0$$

$$\vec{j} = ne\left\langle \vec{V} \right\rangle = ne\frac{e\vec{E}\tau}{m}$$

$$\left\langle \vec{V} \right\rangle = \frac{e\vec{E}\tau}{m}$$

$$\sigma = \frac{ne^2\tau}{m}$$

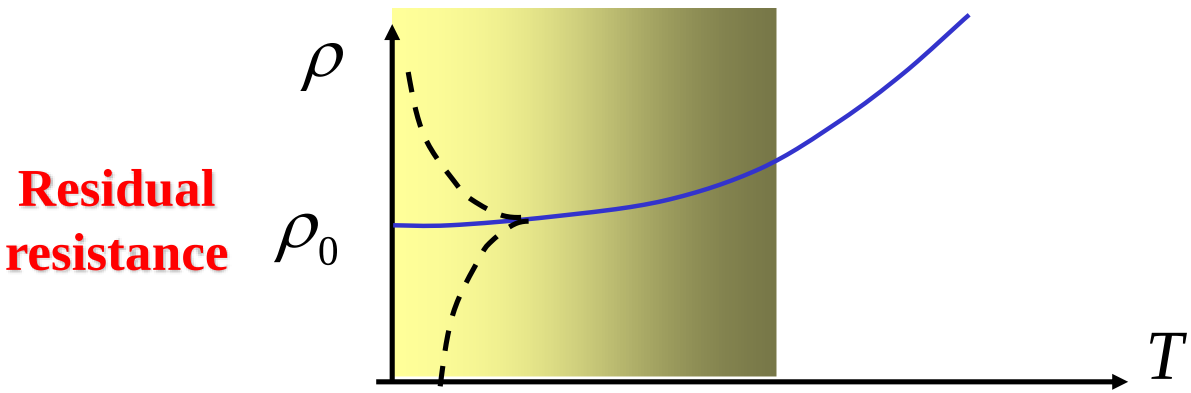
perfect conductor : $\tau \to \infty \qquad \rho \to 0$

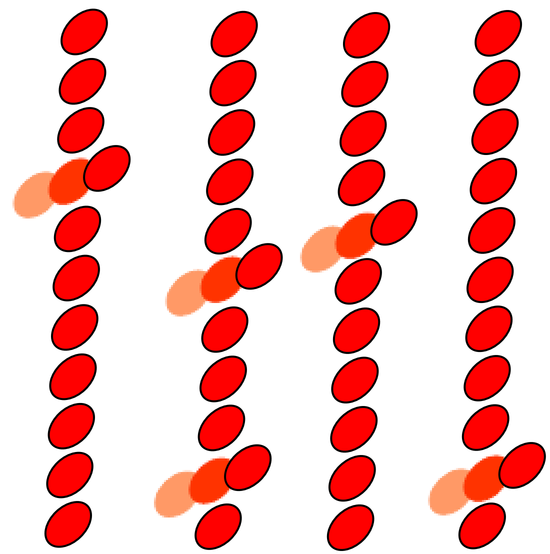
Newton equation with damping:

$$\frac{d\vec{p}}{dt} = e\vec{E} - \frac{\vec{p}}{\tau}$$

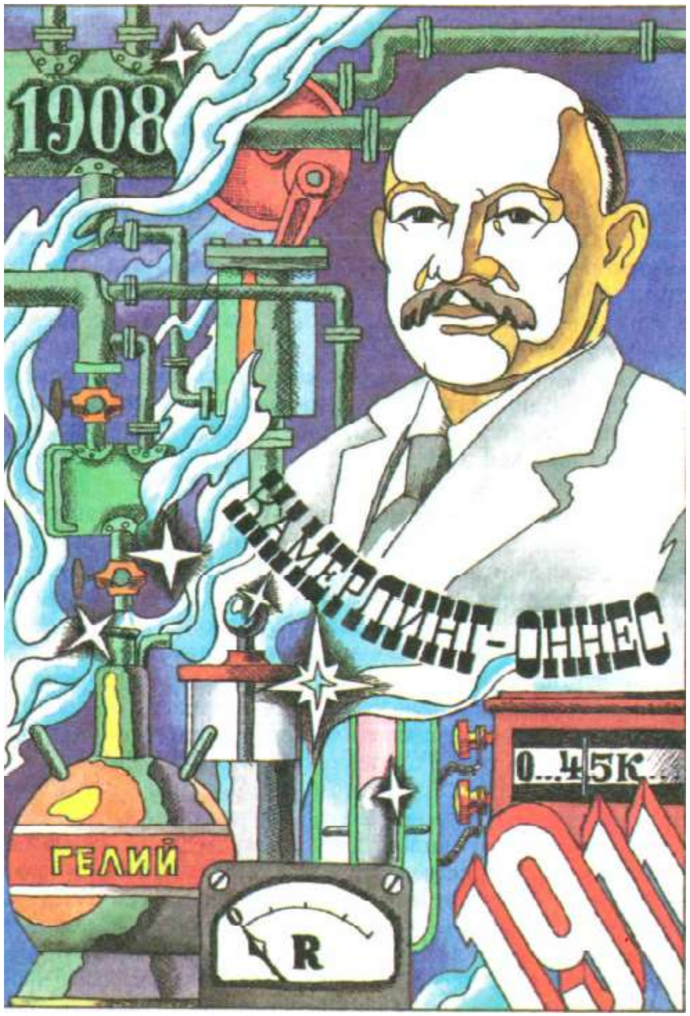
Drude model. **Questions and problems….**

- **What is τ ? What is the cause of electron scattering?**
- **How to define n ?**
- **What happens in perfect crystal and in the presence of defects?**
- **can the ions move?**





Heike Kamerlingh Onnes



On the way to low temperatures
Leiden (1908)

Liquid ^{4}He (T=4,2K)

0 K = -273 C

Mercury (Hg) resistance vs temperature (1908)

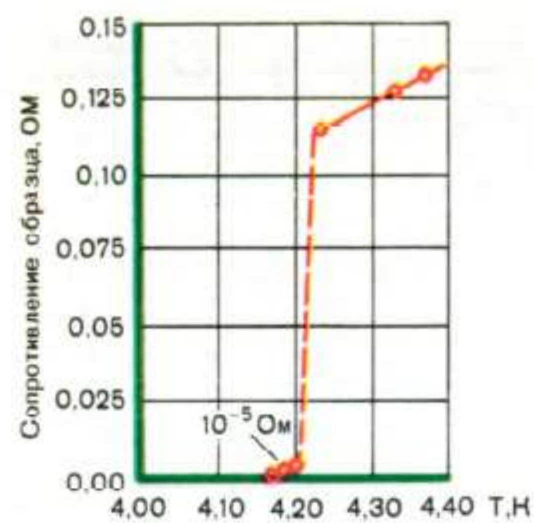


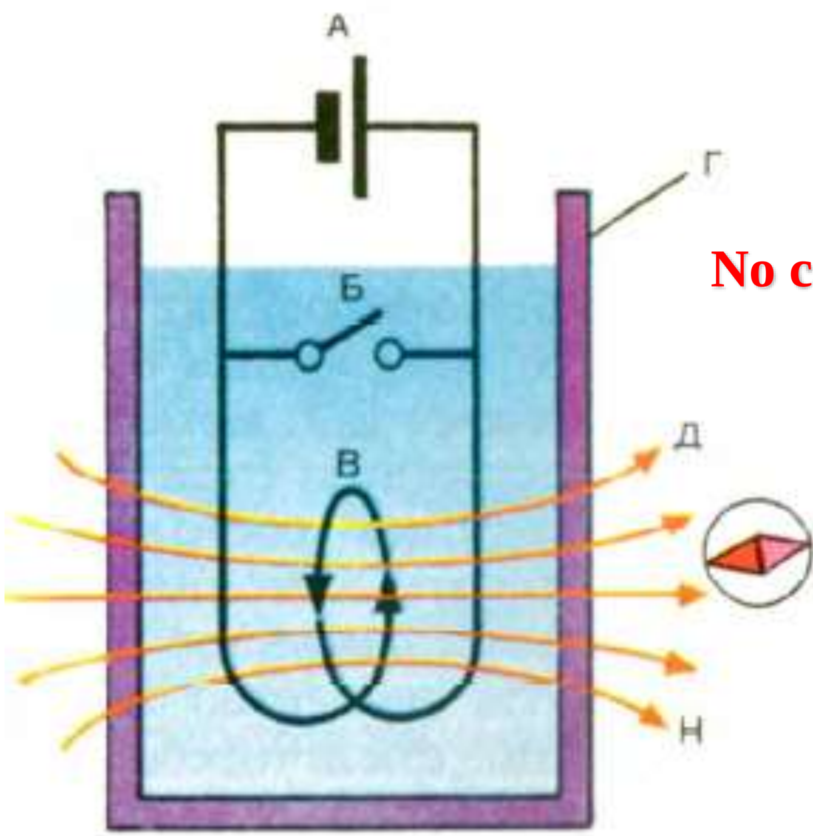
Table 1.1: Parameters for metallic superconductors

	T_c, K	H_c, Oe	H_{c2}, Oe	λ_L, $\AA$	ξ_0, $\AA$	κ	Type
Al	1.18	105		500	16000	0.01	I
Hg	4.15	400		400			I
Nb	9.25	1600	2700	470	390	1.2	II
Pb	7.2	800		390	830	0.47	I
Sn	3.7	305		510	2300	0.15	I
In	3.4	300		400	3000		I
V	5.3	1020		400	~300	~ 0.7	II

Table 1.2: Parameters for some high temperature superconductors

	T_c, K	H_{c2}, T	λ_L, $\AA$	ξ_0, $\AA$	κ	Type
Nb_3Sn	18	25	~2000	115		II
$La_{0.925}Sr_{0.072}CuO_4$	34		1500	20	75	II
$YBa_2Cu_3O_7$	92.4	150	2000	15	140	II
$Bi_2Sr_2Ca_3CuO_{10}$	111					II
$Tl_2Sr_2Ca_2Cu_3O_{10}$	123					II
$HgBa_2Ca_2Cu_3O_8$	133					II
MgB_2	36.7	14	1850	50	40	II

Persistent current in a closed loop



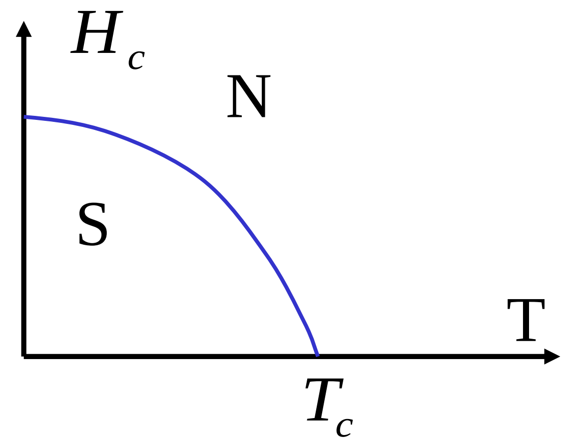
No change of magnetic field

Magnetic arrow

Disappointment: increasing current and magnetic field destroy superconductivity

$$\frac{H_c^2}{8\pi} = f_n - f_s$$

Free energies of normal and superconducting phases

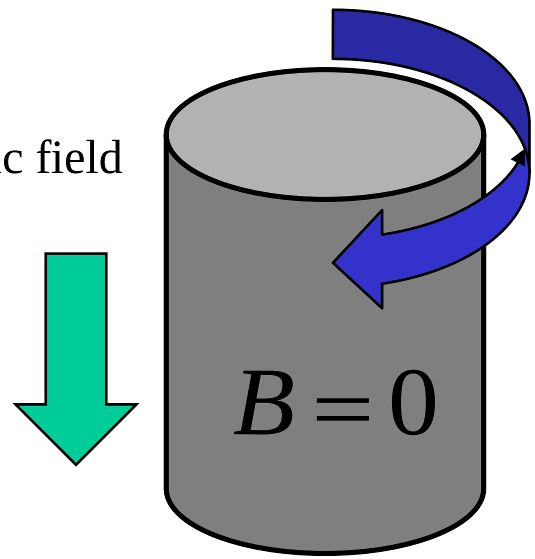


Magnetic properties of superconductors. Meissner – Ochsenfeld effect (1933)

Superconductor. Field cooled or zero field cooled sample

Screening current

Magnetic field H



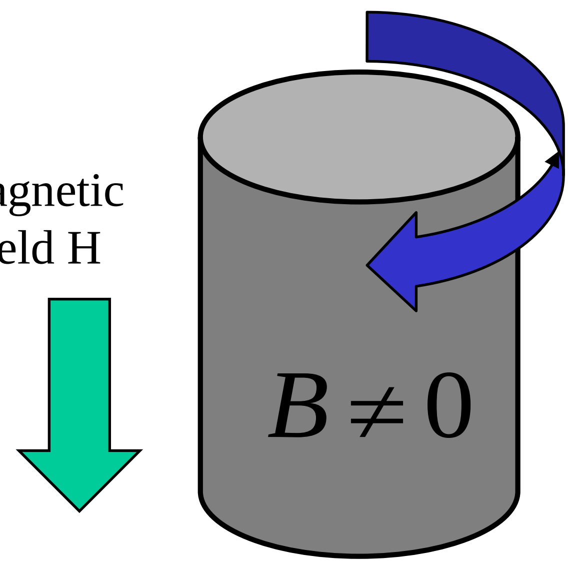
Perfect conductor. Field cooled sample

$$rot\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E}d\vec{\ell} = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$$

$$\vec{E} = \rho\vec{j} = 0 \qquad \Phi = const$$

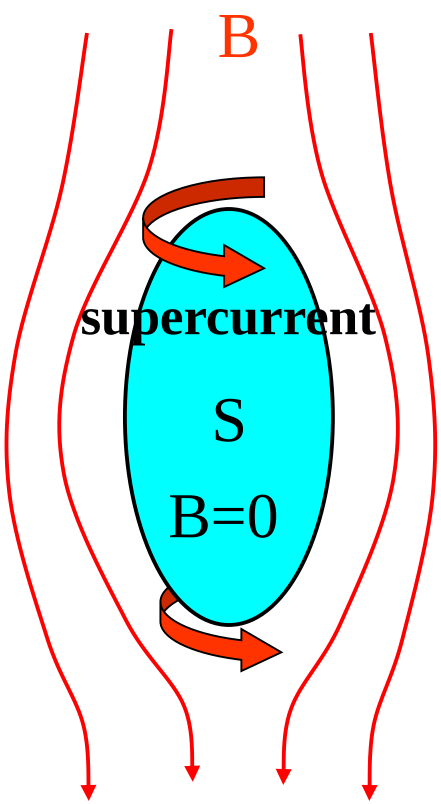
Screeneng current=0

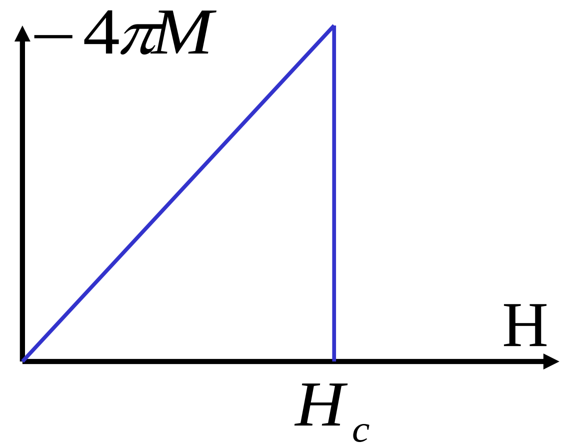
Magnetic field H



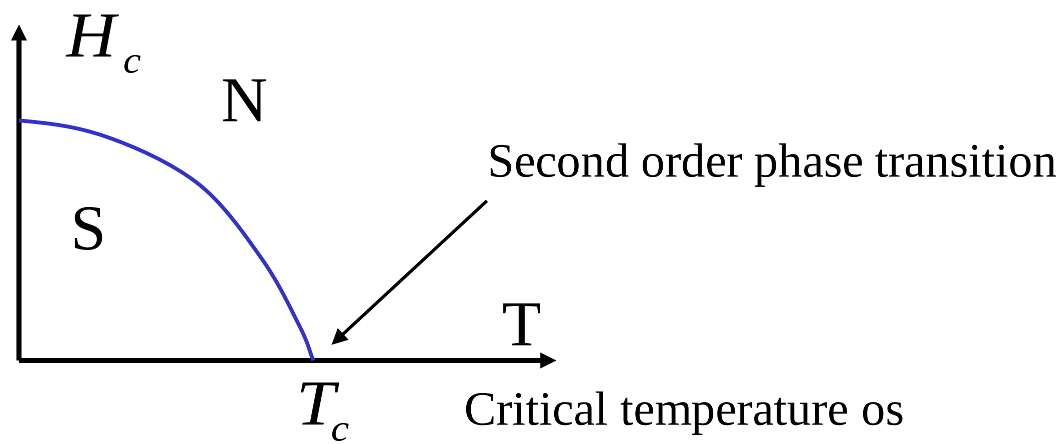
Superconductors in magnetic field. Phase diagram.

Type-I superconductivity





1 order phase transition



Critical temperature os superconducting transition

Levitation

2 variants: magnet levitates above superconductor or vice versa.

problem: explain levitation and find the expression for the force balancing the gravitation force.

Some theoretical exersise. Again Drude model.

$$\frac{d\vec{p}}{dt} = e\vec{E} - \frac{\vec{p}}{\tau}$$

$$\vec{E} = \vec{E}_\omega e^{i\omega t}$$

$$\vec{p} = \vec{p}_\omega e^{i\omega t}$$

$$\vec{p}_\omega = \frac{e\vec{E}_\omega}{i\omega + \tau^{-1}}$$

$$\vec{j}_\omega = ne^2 \frac{\vec{E}_\omega}{m(i\omega + \tau^{-1})}$$

London equation

Current of superconducting electrons:

$$\vec{j}_\omega = \frac{n_s e^2}{im\omega}\vec{E}_\omega \qquad \tau = \infty$$

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

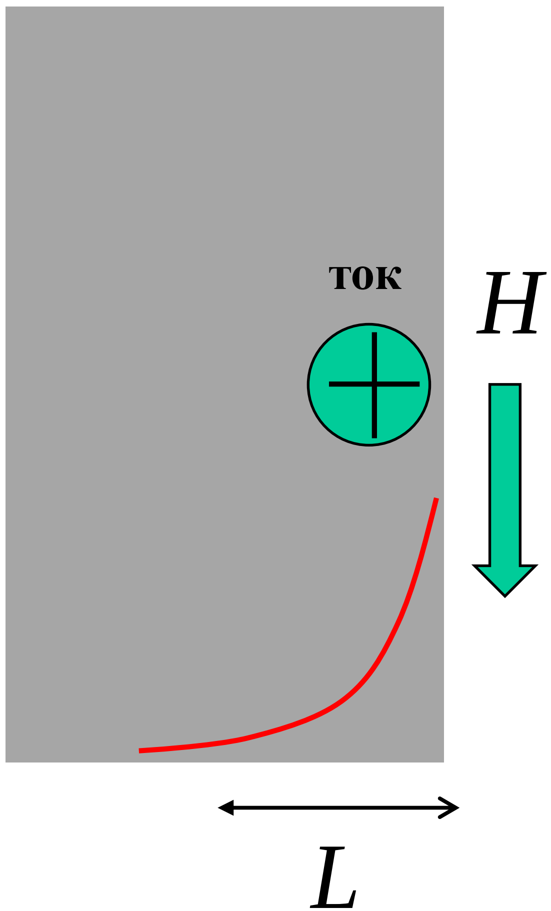
$$\vec{j}_\omega = -\frac{n_s e^2}{mc}\vec{A}_\omega = -\frac{c}{4\pi\lambda^2}\vec{A}_\omega$$

Magnetic field penetration depth

$$\vec{j} = -\frac{c}{4\pi\lambda^2}\vec{A}$$

What about gauge invariance?

superconductor



1. Jump in magnetic field value=surface current

$$jL \sim \frac{c}{\lambda^2} AL \sim cH$$

2. Vector potential

$$A \sim HL$$

$$L \sim \lambda$$

$$\lambda(T=0) \sim 100 - 1000A$$

Magnetic field penetration depth= London penetration depth

$$rot\vec{B} = \frac{4\pi}{c}\vec{j}$$

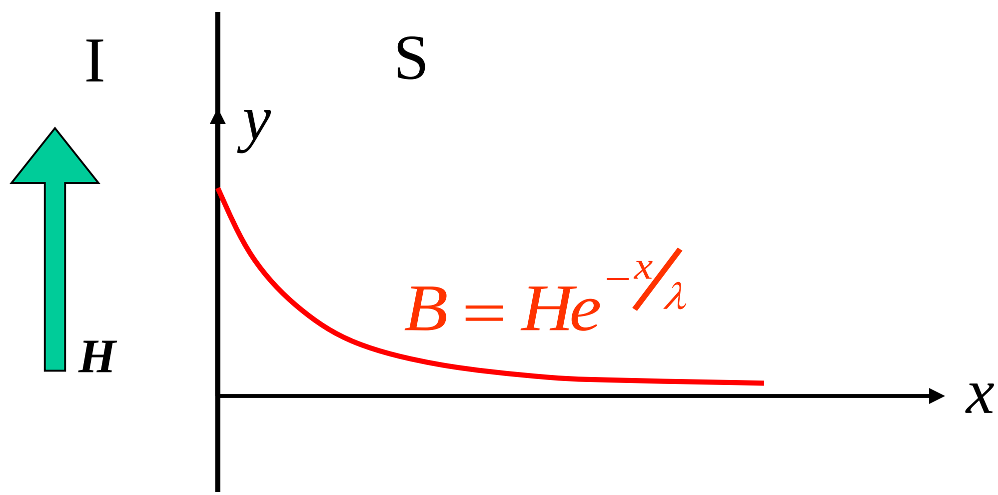
$$rotrot\vec{B} = -\Delta\vec{B} + \nabla div\vec{B}$$

$$\vec{B} = rot\vec{A}$$

$$\vec{B} = B(x)\vec{y}_0$$

$$B(0) = H$$

$$B'' = \frac{1}{\lambda^2}B$$



$$\lambda(0) \sim 10^3\,\mathring{A}$$

Limit of perfect magnetic screening.

$$\lambda \to 0$$

Magnetic field lines outside superconductor are parallel to its surface

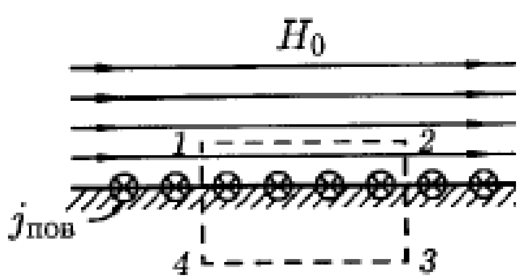
$$div\vec{B} = 0$$

$$rot\vec{B} = \frac{4\pi}{c}\vec{j}$$

$$rot\vec{H} = \frac{4\pi}{c}\vec{j}_{ext}$$

$$\vec{j} = \vec{j}_{ext} + c \cdot rot\vec{M}$$

$$\vec{B} = \vec{H} + 4\pi\vec{M}$$



$$\mathbf{j}_{пов} = \frac{c}{4\pi}[\mathbf{n}, \mathbf{H}_0];$$

Thermodynamic arguments. Critical magnetic fields.

Full diamagnetism

$$\vec{B} = 0$$

$$\vec{H} = -4\pi\vec{M}$$

The work of the source of the magnetic field H

$$-\vec{M}d\vec{H} = \frac{1}{4\pi}\vec{H}d\vec{H}$$

$$-\int_0^H \vec{M}d\vec{H} = \frac{1}{4\pi}\int_0^H \vec{H}d\vec{H} = \frac{H^2}{8\pi}$$

Density of the free energy in the field

$$F_s = f_{s0} + \frac{H^2}{8\pi}$$

Condition of transition to the normal state

$$F_s = f_{s0} + \frac{H_{cm}^2}{8\pi} = f_n$$

Energy of the favorability of superconducting state

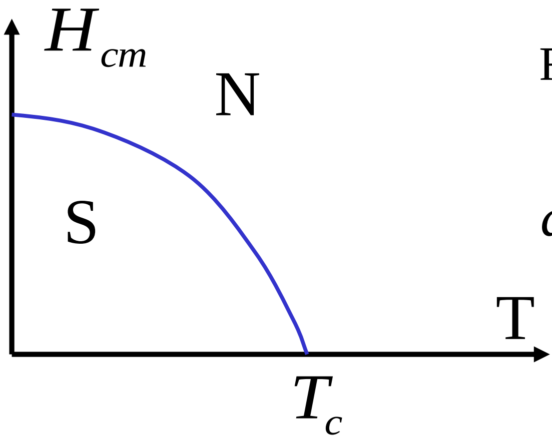
$$\frac{H_{cm}^2}{8\pi} = f_n - f_{s0}$$

Levitation.

$$\vec{f} \sim -\nabla\left(\frac{H^2}{8\pi}\right)$$

?

Is it correct?
What about stability?
Earnshaw's theorem?



Free energy $F = U - TS$

$$dF = dU - TdS - SdT = -pdV - SdT$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$S_n - S_s = -\frac{H_{cm}}{4\pi}\frac{\partial H_{cm}}{\partial T}$$

1. Nernst theorem

$$S(T=0) = 0$$

$$\left.\frac{\partial H_{cm}}{\partial T}\right|_{T=0} = 0$$

2. Superconducting state is more ordered $\frac{\partial H_{cm}}{\partial T} < 0$ $\quad S_n - S_s > 0$

3. Transition at T=Tc is second order phase transition

4. In magnetic field we get 1st order phase transition

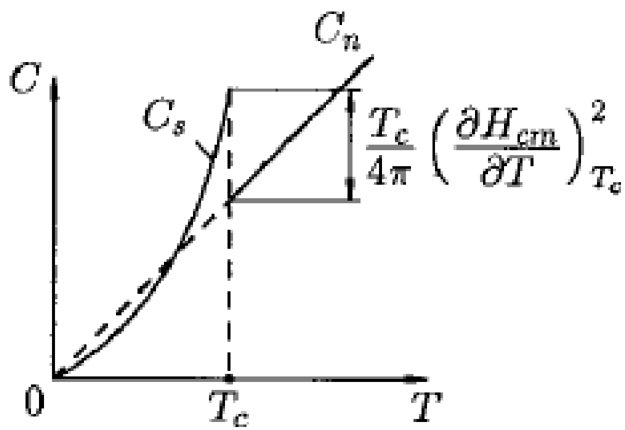
Electronic specific heat

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

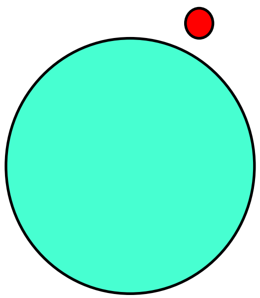
$$S_n - S_s = -\frac{H_{cm}}{4\pi}\frac{\partial H_{cm}}{\partial T}$$

$$C_n - C_s = -T\frac{H_{cm}}{4\pi}\frac{\partial^2 H_{cm}}{\partial T^2} - \frac{T}{4\pi}\left(\frac{\partial H_{cm}}{\partial T}\right)^2$$

$$C_n - C_s = -\frac{T_c}{4\pi}\left(\frac{\partial H_{cm}}{\partial T}\right)^2\Bigg|_{Tc}$$



Electronic specific heat in normal state?



$$U \sim N_F \cdot T \cdot T$$

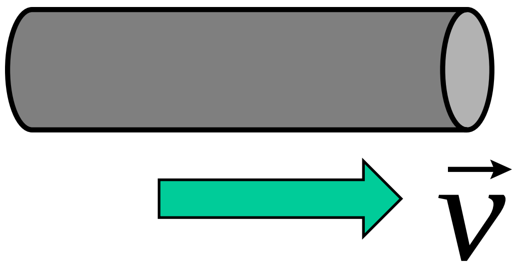
$$C_V = \frac{\partial U}{\partial T} \sim N_F \cdot T$$

Temperature dependence of phononic contribution to specific heat?

Some milestones and history

Superconductivity = superfluidity of electronic fluid?
Landau criterion of superfluidity (1941)

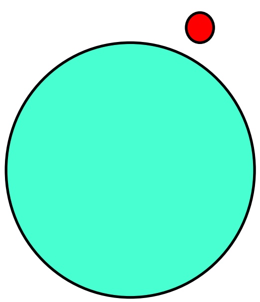
^{4}He at temperatures below 2K in capillary



Energy of excitation which results in dissipation

$$E' = \varepsilon(p)$$

$$E = \varepsilon(p) + \vec{p}\vec{v} < 0 \qquad v > \min \frac{\varepsilon(p)}{p} = v_c$$



Fermi sphere

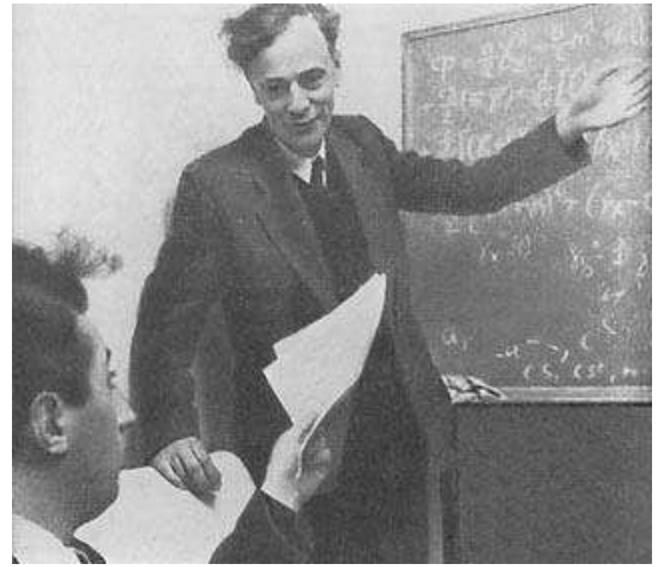
Q. No minimum quasiparticle energy in metals!?

A. To get the energy gap electrons should form pairs

Phenomenological Ginzburg – Landau theory (1950).

Quantum mechanics at macroscale.

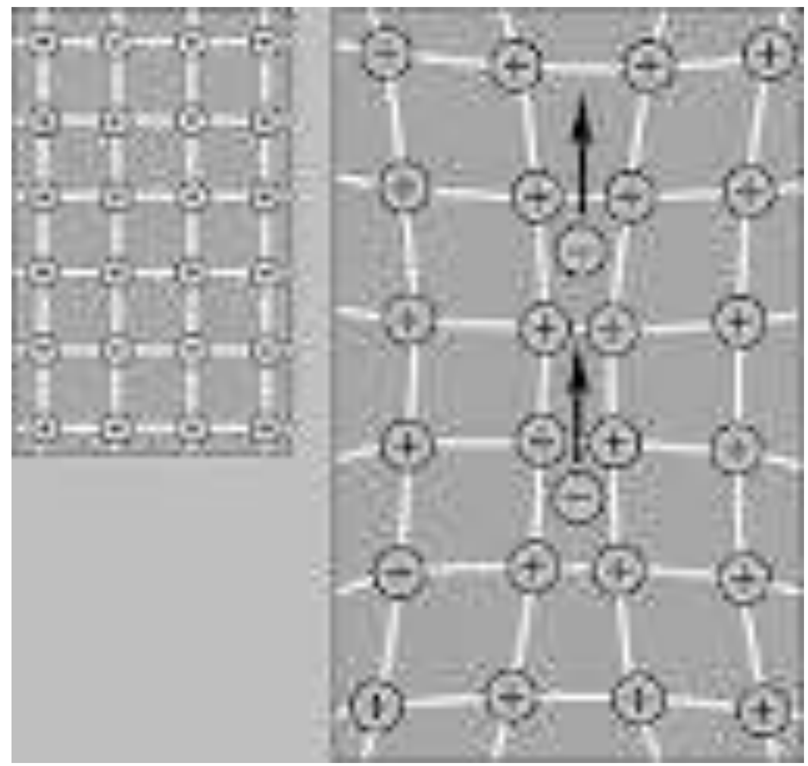
Nobel prize -2003



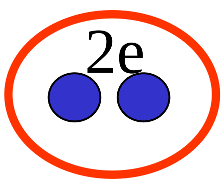
$$\Psi$$

Microscopic theory of superconductivity 1957- BCS
Bardeen – Cooper - Schrieffer

Size of the Cooper pair:
100-1000 A



Attraction of electrons results in formation of bound electron pairs



BCS expression for superconducting critical temperature

$$T_c = \omega_D e^{-\frac{1}{gN}}$$

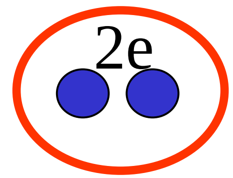
Isotope effect

$$\omega_D \propto \frac{1}{M^{1/2}}$$

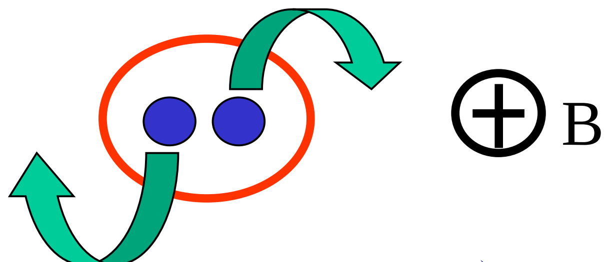
$$M\frac{d^2x}{dt^2} + kx = 0$$

Why does the magnetic field destroy superconductivity?

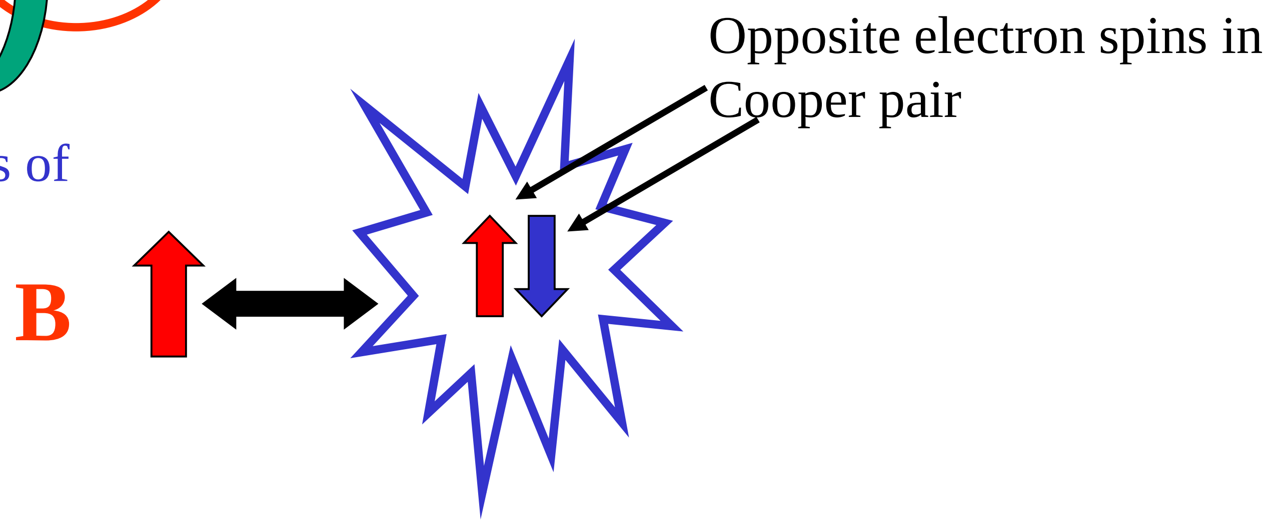
Mechnisms of interaction of the magnetic field with the Cooper pair



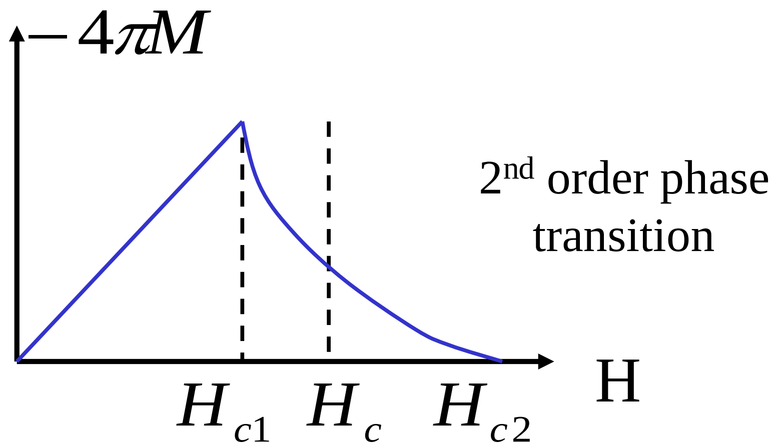
- orbital (electromagnetic) mechanism

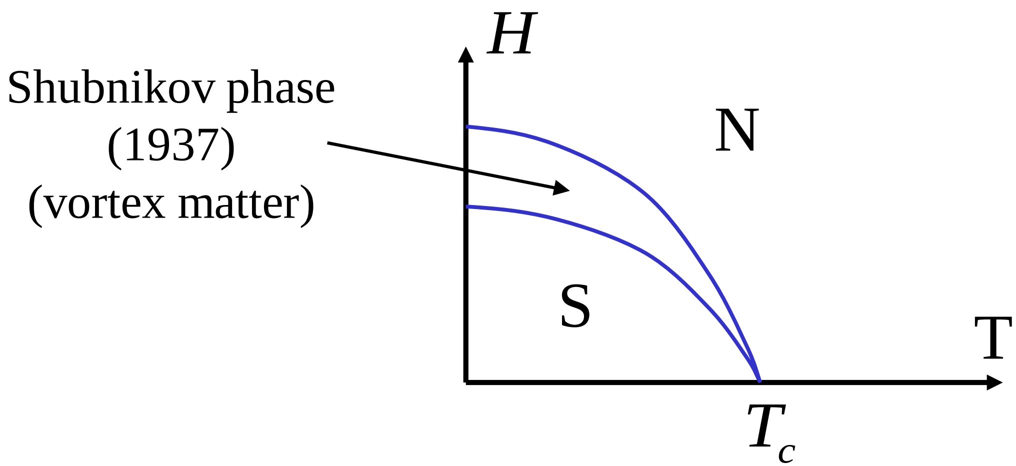


- interaction with the spins of electrons



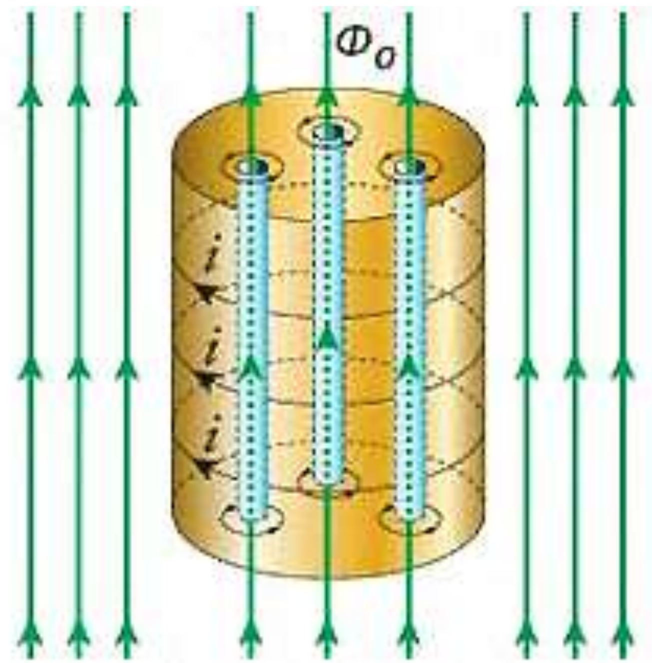
Type-II superconductivity. Alloys. High critical fields and currents





Type – iI superconductivity. Abrikosov vortices – tubes of magnetic flux.

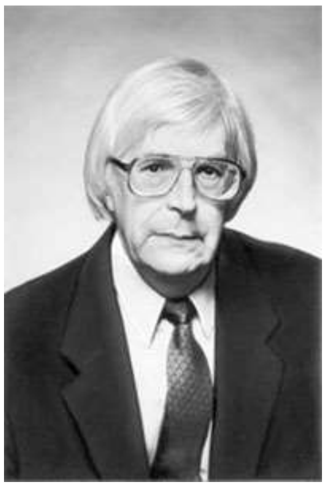
Nobel prize -2003



More milestones of theory

N.N.Bogolubov

Equations for interacting electrons and holes in superconductors 1958- 1959



Gorkov equations 1958

Andreev reflection 1964

Applications of superconductivity

No dissipation:

Strong magnets,
Energy transmission,
Superconducting cables

Рис.7. Сверхпроводящий соленоид (весом 224 т, включая около 15 т Nb-Ti сверхпроводящего кабеля) для детекторов частиц Большого адронного коллайдера

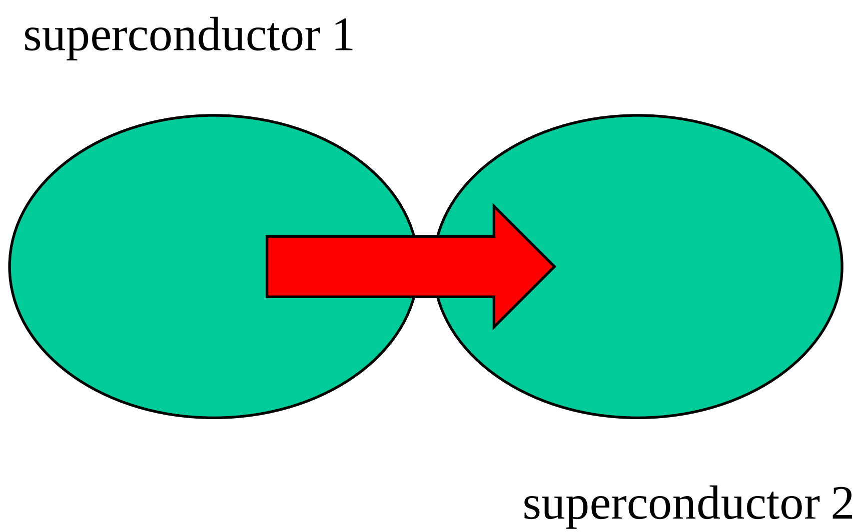
applications

Meissner effect, levitation: transport

applications

Flux quantization,
Josephson effect:

Magnetic field sensors,
SQUIDs,
Medicine,
Cryoelectronics,
Photon detectors
Quantum computing …



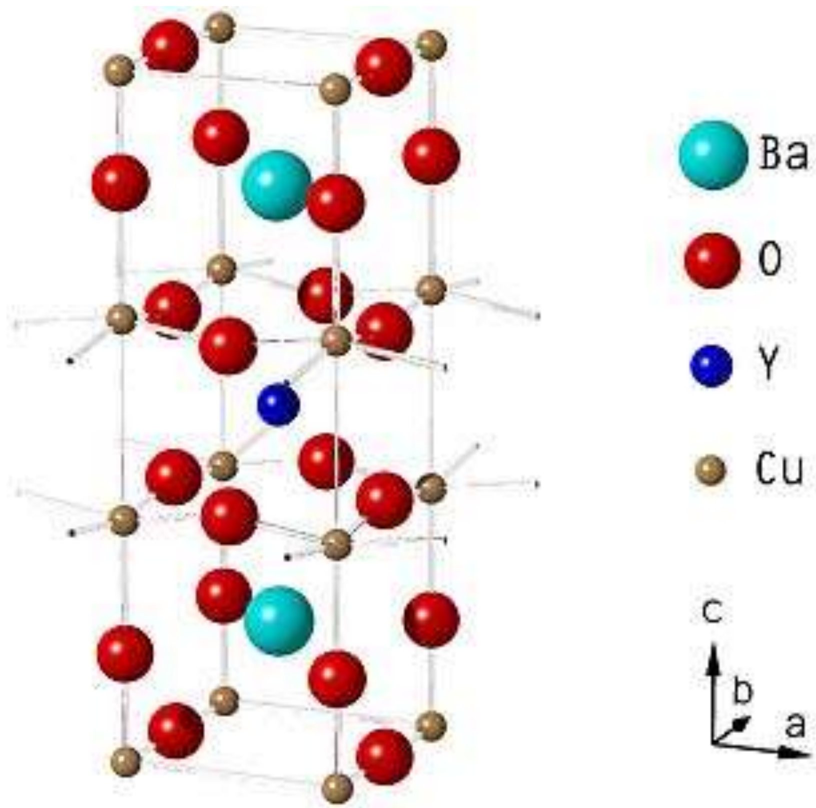
Q. Can we get a supercurrent between 1 and 2?

A. Yes, it is called Josephson effect

Race for high- temperature superconductivity.

before 1986г. – no big progress – maximum critical temperature~ 24К (Nb3Ge)

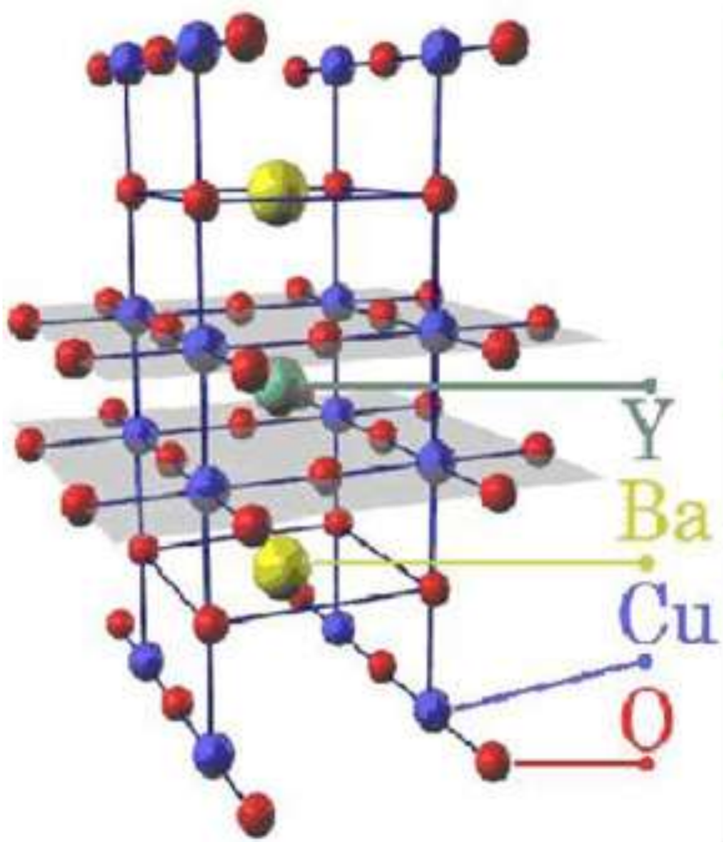
Bednorz, Muller (1986)

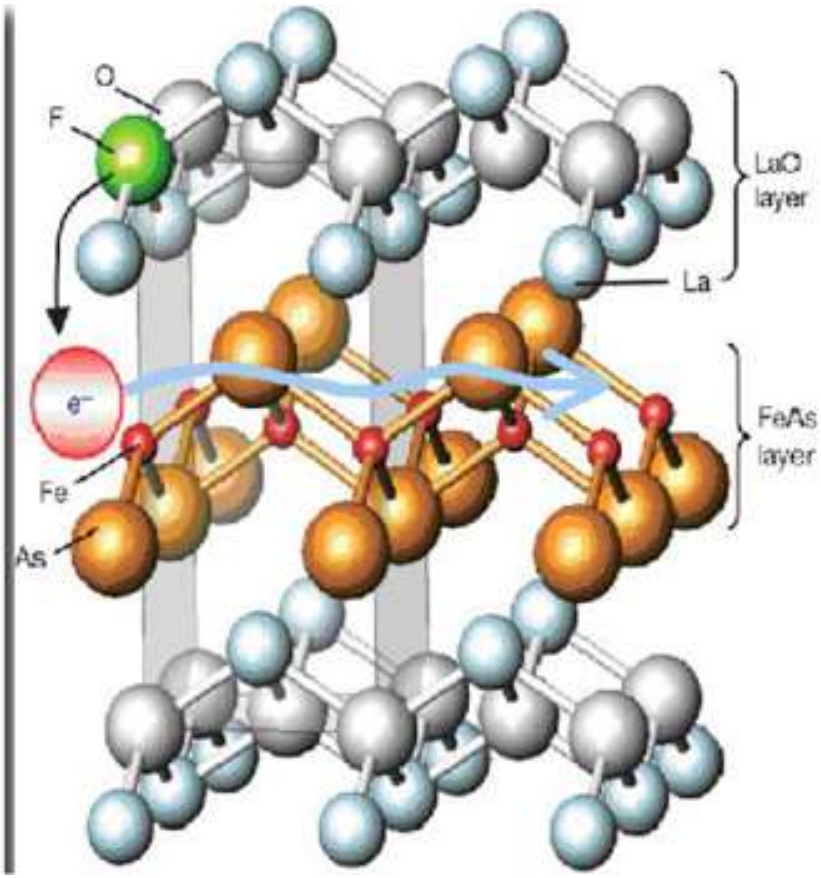


Jump above liquid nitrogen temperature = 77K

New classes of superconductors

cuprates





Fe based compounds

Questions for home work:

Is there any upper limit for superconducting critical temperature?

Do we really need attraction between electrons to get superconductivity?

Suggest new version of application?